A General Framework for International Conflict under Incomplete Information

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Abstract

The crisis bargaining literature identifies the incentives and constraints that states face in an international conflict. It shows that the size of and the relation between economic cost resulting from engaging in a coercion and domestic audience cost resulting from issuing empty threats determines states' prospects to succeed at the threat stage and engage in coercion. This article further develops the crisis bargaining framework by providing a specification of uncertainty about the economic and domestic audience cost. Specifically, we introduce uncertainty by modelling economic and domestic audience costs as probability distributions, instead of fixed values as in earlier studies. We develop our argument with the use of a game theory model, expanding the current crisis bargaining framework. In contrast to earlier work, we find that an increase in uncertainty decreases the effectiveness of threats of coercion and decreases the probability of engagement in coercion.

1 Introduction

Conflict between states comes at a great cost (Pinker, 2011; Goldstein, 2011; Pond, 2017; Neuenkirch and Neumeier, 2016) and the fact that states fail to agree to a settlement before engaging in coercion is a puzzle (Fearon, 1995) motivating extensive research into the conditions for the onset of and success in war or economic sanctions (Bas and Schub, 2018; Gartzke et al., 2017). To this end, scholars have developed a number of theories to understand how conflict in international relations unfolds, focusing on the role of economist cost, audience cost, uncertainty and signalling (Bas and Schub, 2018; Gartzke et al., 2017). In this article, we build on this literature and offer a unified framework, with a complex understanding of uncertainty, clear specification of the role of signalling and sharp empirical predictions. Effectively, this article answers the call for "greater precision" (Bas and Schub, 2018) in research on conflict in international relations and creates a unified platform on which further empirical and theoretical work can develop.

In this article, we propose a crisis bargaining model, in which the probability of success of a threat and engagement in a coercion is not only influenced by domestic audience and economic cost, but also by the uncertainty in a state's belief about the costs and benefits of the opposing party. We make these costs and benefits and accompanying probabilities explicit, which allows for application of the model to various conflict scenarios. Our model contributes to existing literature in two ways. First, we offer a game theory crisis bargaining model and derive success and engagement probabilities formally, which allows for easy application to various conflict scenarios, whereas the current scholarship does not specify any type of probabilities. We explore the model outcomes across various values of the parameters, leading to a number of propositions on the effects of economic costs, audience costs and payoff uncertainty on the conflict's outcome. Second, we distinguish the constraining role of information and domestic audience cost as two separate effects in determining a situation's outcome; whereas previous literature did not differentiate between domestic audience cost and information, assuming it to be a single mechanism. In addition, we make the code of all our analyses available in a user-friendly and well-documented archive, thereby inviting other researchers to adjust our baseline conditions and generate own results with our crisis bargaining model.

Our paper is structured as follows. First, in Section 2, we offer an overview of the literature on crisis bargaining and list the limitations of the current approaches. Then, we develop a novel game theory model for crisis bargaining under complete and incomplete information in Section 3. Next, in Section 4, we analyse the prediction of our model in respect to effectiveness of threats and onset of coercion; we also derive a number of propositions. Finally, we conclude the article in Section 5.

2 Literature review

2.1 Conflict in International Relations

Fearon (1995) highlights that "under broad conditions the fact that fighting is costly and risky implies that there should exist negotiated agreements that rationally led states in dispute would prefer to war"; yet, we continuously observe states engaging in conflict. For scholars there are two explanations for this phenomenon consistent with states rationality — namely, uncertainty and commitment problem (Fearon, 1995). The former causal argument, uncertainty, has received extensive scholarly attention, with three sources of uncertainty identified as core: asymmetric information about the type of player, asymmetric information about the intention of the player and fundamental uncertainty related to the stochastic nature of international relations (Bas and Schub, 2018).

The first source of uncertainty in conflict in international relations, asymmetric information about players' type, is the most widely studied (Bas and Schub, 2018; Gartzke et al., 2017). It is focused on the features of the payoff matrix that actors base their choices upon. Here, the cost of war, in the absolute and relative sense, is seen as key determinant of states' action (Powell, 1988; Fearon, 1995; Schultz, 1999; Kydd, 2003; Wolford et al., 2011). Low costs of war are listed as a characteristics of "high resolve" states — actors likely to eventually engage in conflict. On the other hand states with high costs of war are "low resolve", as they are not likely to engage in conflict. Cost of war is also a relevant feature of states' decision-making about engaging in war in relative terms, as players weight the cost of war with settling with audience cost, if a threat of conflict has been issued but not acted upon buy the sender and, consequently, the sender will be penalised by the voters for not keeping up the promise (Whang et al., 2013; Schultz, 1999; Kertzer and Brutger, 2016; Fearon, 1994). Thus, here the source of uncertainty can be two-fold: the absolute cost of war and the relative cost of war to the domestic audience cost. What is more, states have major incentives to exploit this uncertainty and potentially win concessions that are otherwise unattainable. As a result, we observe that "low resolve states have incentives to claim that they are indeed high resolve states. Unable to distinguish between types of opponents, proposing states balance a 'risk-return tradeoff,' which can lead to war." (Bas and Schub, 2018). A closely related theoretical argument to the cost of war is the role of relative capabilities, where states wage the decision of war based on the military capabilities of their own and of the opponent. A number of scholars point to the uncertainty about relative capabilities as a key source of conflict on international relations (Blainey, 1988; Morrow, 1989; Wagner, 1994; Van Evera, 1999; Smith and Stam, 2004; Johnson, 2004; Slantchev and Tarar, 2011). Furthermore, scholars indicate that uncertainty about states' ability to deploy effectively their capability may be a more relevant source of uncertainty than the absolute capacity of the opponent itself (Arena, 2013).

Research indicates two key mechanisms that address the problem of uncertainty in respect to player type — and by the same token help to avoid conflict between states. Tying hands and sinking costs are listed, under the umbrella of costly signalling, as mechanisms that allow states to address the problem of asymmetric information (Gartzke et al., 2017; Bas and Schub, 2018). Tying hands relates to an action of a state, where the player binds itself to bare a cost in a future if a particular decision path is followed. For example, a political leader may experience a domestic audience cost for not following up on a threat of economic sanctions, had the target state not comply with the threat and adjust its policy (e.g., Drezner, 2003; Whang et al., 2013). This theoretical mechanism directly relates to the previously discussed problem of player's type, resolve and relative cost of war. A player, by tying hands, can signal that it will choose to engage in coercion if necessary, as the domestic audience costs are larger than than cost of coercion and a rational actor is better off engaging in, for example, war or economic sanctions (Schultz, 1999; Fearon, 1997; Kertzer and Brutger, 2016). The second mechanism, sinking costs, appears more strongly linked to address the uncertainty related absolute costs of coercion and to relative power. It also directly affect the relative power balance. Sinking costs are efforts of a player before the conflict escalates to indicate readiness to fight, for example troop mobilisation. This can directly translate to lower costs of war, once conflict escalates, as part of the associated costs are already foregone — hence the name, sunk costs (Fearon, 1997). In general, tying hands is an action listed as a signal costly *ex post* — for example, an empty threat of economic sanctions will only be (domestic audience) costly if not followed, not at the time of issuing. Sinking costs are signals costly *ex ante*, as a state has to incur the costs before the conflict unfolds and may never materialise (Gartzke et al., 2017).

Current scholarship raises a number of criticism on the theoretical underpinnings of the study of uncertainty and conflict in international relations. To begin with, the theory does not offer clear empirical predictions "linking uncertainty and the opponent's actual type to conflict probability" (Bas and Schub, 2018). Second,the theory does not guide researchers on how to approach the *ex ante* opponents type — before a costly signal has been issued. Third, the theory is not clear, nor consistent, on how the gradation in uncertainty affects conflict outcomes in international relation. Finally, the theory is not clear on whether empirical researchers should seek for a proxy for uncertainty or try to approximate the player's type (Bas and Schub, 2018). Not surprisingly, given the lack of clear guidance offered by theory on the role of uncertainty in conflict, empirical research on the mechanisms to address uncertainty — hands tying and sinking costs — are inconclusive (Gartzke, 2007; Kertzer and Brutger, 2016; Partell and Palmer, 1999; Schultz, 2001; Snyder and Borghard, 2011; Trachtenberg, 2012)

In the next subsection, we zoom into the details of the game theory models that are developed based on the theoretical concepts and mechanism related to uncertainty, signalling and conflict in international relations. The role of mathematical models is to streamline the theoretical concepts into logically consistent representation of state behaviour and generate empirically testable insights.

2.2 Formal Model of Conflict

A common way to formalise and give rigour to the theoretical concepts related to the rational-choice approach to uncertainty and conflict in international relations is to employ game theory (e.g., Drezner, 2003; Schultz, 1999; Lacy and Niou, 2004). In such a game, conflict is treated as a sequential game. The two players in a situation of conflict, the sender and the target, decide sequentially whether to (1) issue a threat of coercion (sender's turn), (2) resist the threat of coercion if a threat is issued by the sender (target's turn), and (3) follow up on the threat with coercion if the target resists (sender's turn). The sequential nature of these models leads to the concept of a 'game tree', consisting of several layers (turns), with various ending branches (outcomes). The goal of these game theory models is to pinpoint the relation between economic and domestic audience costs, clarify the role of uncertainty and, eventually, to identify under what conditions threats succeed and coercion engagement occurs. We list a number of common assumptions and characteristics among these game theory models of crisis bargaining in the following, based on the work of Fearon (1994); Drezner (2003); Schultz (1999); Signorino (1999); Whang et al. (2013) and Lacy and Niou (2004) and link it to the broader theory of uncertainty and conflict in international relations (Bas and Schub, 2018; Gartzke et al., 2017):

- The sender and target are assumed to play a game of incomplete information, meaning that the payoffs at each instance of the game are private information — actors do know their own payoffs, but do not know the exact payoffs of the opponent. This reflects one source of uncertainty that stimulates conflict between states: "asymmetric information about adversary traits" that influence payoffs in a conflict situation (Bas and Schub, 2018).[1]

- The sender and target are assumed to know a 'mean' payoffs of the opponent, where 'mean' relates to the costs and benefits averaged over all potential conflict that the sender could face. In other words, the target and the sender do not know what are the exact costs for their conflict and operate under a form of 'common knowledge' available to all actors.

- The domestic audience cost and economic cost for engaging in coercion are assumed to have a uniform probability distribution (between 0 and 1); thus, each payoff that is possible is equally likely. Nevertheless, the exact probabilities are not specified in the literature.

- Scholars assume a (negative) domestic audience cost as a penalty for issuing an empty threat and (non-positive) economic costs for engaging in coercion.

- The sender is assumed to only follow-up on a threat if she expects that the cost of engaging in a coercive action is smaller than the domestic audience cost, highlighting the role of the

[1] Yet in the models, only the costs of conflict is treated as unknown — the audience costs for backing down (sender) or concession to a threat (target) are considered known.

relative cost of conflict and the audience cost. Related to this, scholars assume that the 'mean' domestic audience costs increase with the level of democracy of the sender state.

- Finally, audience costs (partly) determine the 'resolve' of the sender. High resolve senders have higher domestic audience cost for backing down on a threat relative to economic cost for engaging in a coercion. And the opposite holds for low resolve players. Consequently, high resolve players follow up, engaging in coercion after a threat if the target stands firm. Low resolve players issue empty threats and do not follow with engaging in coercion.[2]

2.3 Contribution to the literature

In the following section of this article, we offer a game theory model of conflict in international relations that builds both on the literature on uncertainty and conflict and on the formal models of sequential and incomplete information games developed to formalise this theory.

We do this by offering an improved model of conflict bargaining, with a complete specification of the role of costs (domestic audience cost and economic cost of conflict) and uncertainty. First, we formalise uncertainty on all the costs faced by the sender and the target. Second, our model focuses on the more fundamental quantity of cost of engaging in coercion, and allows one to calculate success probabilities based on these costs. Finally, we distinguish uncertainty as a separate mechanism, next to economic and domestic audience cost, in determining the outcome of the game.

Our model contributes in two important ways to the current scholarship on conflict in international relations. First, it produce clear empirical predictions in respect to the effectiveness of threats and the prospects of conflict onset. Second, it offers a single intuitive framework that incorporates both the sources of uncertainty (information asymmetry and fundamental uncertainty) and means that state use to address this problem (sinking costs and hands-tying). Furthermore, it account for the gradation in uncertainty and costly signalling.

3 A Model for complete and incomplete information

3.1 Conflict bargaining model

In this section, we present the general structure of a conflict bargaining game between a sender and a target. The sender is a state that issues a threat of war or economic sanctions, while the target is a state towards which this threat is directed. For simplicity, we model the sender and the target as unitary actors and assume that the sender engages only in unilateral conflict. The game consists of five phases, in three of which one of the two players has to choose an action. The game is sequential — in each phase only one of the two players chooses an action, based on the action of their opponent in the previous phase.

[2] A strategy referred in the literature as bluffing, see Whang et al. (2013)

Let us assume a scenario where an issue appears between two states (A and B for simplicity), for example an uranium enrichment programme in state B that is highly criticised by state A. This programme is very important to state B and its continuation produces positive payoff for state B (e.g. medical, energy security and warfare benefits). At the same time it yields a negative payoff for state A (e.g. security concerns). We can depict this situation (and comparable conflicts between states in international relations) as a sequential game between two actors, where the players decide whether or not to threat with a coercive measure, give in to such a threat and implement the threat if necessary. These decisions may be based on both known and unknown payoffs — this is discussed from 3.2 and on — introducing uncertainty of one state in estimating another. We distinguish this uncertainty from signalling, which is discussed in 3.3. The literature typically refers to states A and B as the *Sender* and the *Target* state (of coercion), respectively, and we follow this in our article.

A visual illustration of the framework and associated probabilities and payoffs is shown in Figure 1. We proceed by discussing every phase of this game one by one.

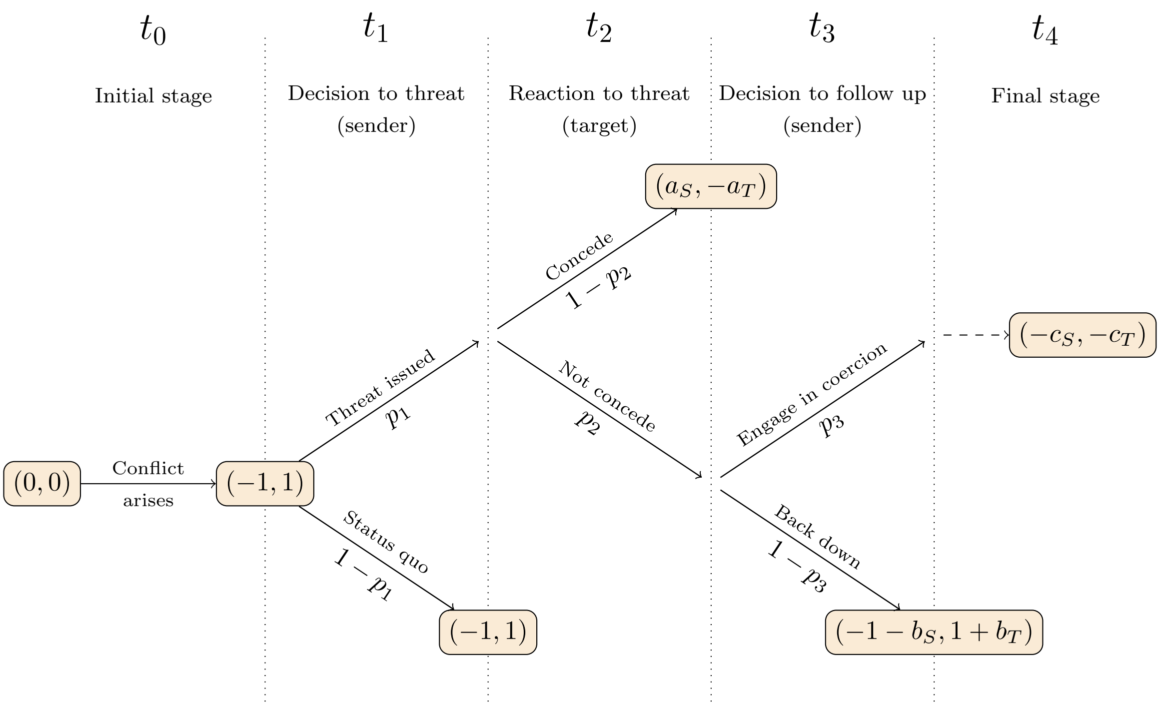

Figure 1. The game consists of five phases (t_0 to t_4). The phases are sequential and the phase's active player is indicated in brackets. The boxed tuples indicate the payoffs in the order of ($Sender, Target$). The payoff parameters are the domestic audience cost a_S and a_T due to the target's concession to a threat (audience benefit for the sender), the domestic audience cost b_S and b_T for the sender's backing down after the threat (i.e., audience 'benefit' for the target) and two cost terms c_S and c_T reflecting the economic costs resulting from engaging in coercion. The probabilities stated underneath every branch are discussed in Section 3.4.

Phase t_0 - Initial stage

The start of phase t_0 represents the baseline phase before a contested policy event occurred. No player takes any action within the game tree, i.e., there is no 'game' element in this phase. With external reasons (not related to the game tree), the policy issue appears, with a positive effect on the target and a negative effect on the sender. The resulting payoffs are marked by $(-1, 1)$: a negative (-1) payoff for the sender, and a positive (1) payoff for the target. The values -1 and 1 are taken for simplicity.

Phase t_1 - Decision to threat (sender's turn)

In phase t_1, the sender needs to decide how to address the policy issue. If she decides to settle with the status quo, the game ends with the same payoffs as phase t_1 started with: $(-1, 1)$. The alternative is to issue a (economic or military) threat of coercion, which continues the game to phase t_2. This stage engages with the concept of hands-tying (Fearon, 1997). Here, the sender commits herself to the domestic audience cost, if backed down on the threat.

Phase t_2 - Reaction to threat (target's turn)

The target state responds to a threat in phase t_2, either conceding to it or not. If the target concedes, the payoffs do not return to the initial $(0, 0)$ in t_0, because the target faces a domestic audience cost of $-a_T$ for abandoning its policy and the sender receives a domestic audience benefit of $+a_S$ for a foreign policy success. If the target state decides *not* to concede, the game enters phase t_3.

Phase t_3 - Decision to follow up (sender's turn)

Now, the sender state has to decide whether or not to follow up on the threat and engage in coercion. If she backs down, the game ends, returning the initial issue to the status quo $(-1, 1)$ and adding a domestic audience cost $-b_S$ for the sender, penalising issuing an empty threat, and a domestic audience benefit $+b_T$ for the target for standing firm in face of foreign pressure.

Phase t_4 - Final stage

If the sender state decides to engage in coercion, the game continues and ends in phase t_4. We parametrise the coercion cost with (c_S, c_T), which are a combination of the eventual conflict outcome (status quo or not), domestic audience costs and economic cost of war or economic sanctions (or both). We assume that the costs of engagement in a coercion are large, so that $c_S \gg a_S, b_S$ and $c_T \gg a_T, b_T$. What is more, this is the stage where the concept of sinking costs enters the game theory model. Sinking costs is a strategy where the sender state can decrease the coercion cost; yet it occurs *ex ante*, before players move on the decision tree.

3.2 Dynamics under complete information

We start by analysing possible dynamics of the crisis bargaining model, under the assumption that each player has complete information about the payoffs (i.e. all payoffs are known) of both oneself and the opponent.

Knowing that all parties are badly affected by engaging in coercion (i.e., economic costs $c_S, c_T \gg 1$), we conclude that threats will only be issued if the sender knows that the target concedes in t_2. The reasoning is as follows, starting from the final stage t_4 and working backwards. If the game ends up in this phase, a sequence of decisions (after the initiation of the game via a threat) must have occurred: a threat was issued, the target did not concede to the threat, and the sender followed up on the threat, engaging the coercion. In such a case, as all payoffs are known, the target must have known in t_2 that the sender would not back down in t_3. Because the target is better off conceding in t_2 (as $-a_T \gg -c_T$), the target would never let the game progress to t_3 (and t_4), so she will always concede in t_2. Analogously, the sender is better off with the status quo than either the outcomes of backing down (t_3) (as $-1 > -1 - b_S$) or engaging in coercion (t_4) as (as $-1 \gg -c_T$), so she will only issue a threat in t_1 if she is sure that the target will concede in t_2.

Hence, the complete information game never reaches t_3 or t_4 and either ends with the status quo, or a successful threat (i.e., target concedes to threat). That is, states never engage in coercion and targets never resists a threat of coercion. Of course, this is not what is observed in reality — states do go to war or impose economic sanctions and threats of coercion are resisted[3]. Still, the complete information game offers us a reference point 'as with all comparative-static exercises, though, the important results deal with the relative probabilities of an outcome across two games, not the absolute probability in either game' (Schultz, 1999). Next, we further expand our theoretical framework and in the following section we relax the assumption of complete information.

3.3 Uncertainty and signalling

In contrast to the previous section, now we study the crisis bargaining model *without* the assumption that both parties know all the payoffs. Uncertainty and beliefs different from the actual payoffs can take various forms and are related to the concept of signalling. We start with the earlier used example of the two states A and B, where state B starts its uranium enrichment programme. State A has to decide whether or not to issue a threat and bases this decision on what she estimates to be all the payoffs: if she thinks that state B will probably concede to the threat, such a threat might be very successful. In reality, however, state A does not posses all the information and considerations of state B, so she will not have a fully accurate estimate of state B's payoffs. This is what we refer to as *uncertainty* of a payoff — the size of a certain non-zero confidence interval around the true value of the payoff, from which state A draws her guess.

Knowing that state B bases its beliefs on state A's payoffs partly in the behaviour of the latter, the action of issuing a threat may alter state B's estimates of the payoffs. In particular, state A might issue a threat with the specific purpose of altering state B's beliefs. This is referred to in

[3]This can be concluded from, e.g., the TIES data set, which indicates a substantial number of cases of failed threats of economic sanctions (Morgan et al., 2014).

literature as *signalling* and, although related, is *inherently* different from uncertainty. Signalling is purposely changing the other state's beliefs by issuing a threat, while uncertainty refers to an ignorance surrounding any payoff as estimated by the other party. In fact, uncertainty can — and is likely to — exist before and after the beliefs of state B are impacted by a signal in a form of threat of coercion.

In terms of modelling, a simple and common method of introducing uncertainty for any unknown variable is by assuming the beliefs of this variable to be Gaussian distributed around the true value. In other words, each unknown variable (i.e., payoff) x is modelled by drawing from $N(\bar{x}, \sigma_x)$, where $\bar{x}$ is the mean value of x and σ_x is the standard deviation. In this article, if not stated otherwise, the true value is used as mean and the standard deviation is taken equal to 1. The width of this normal distribution (modulated by σ_x) is referred to as the uncertainty of x. The uni-modal structure of the Gaussian distributions is chosen deliberately to reflect the assumption that payoff estimates are generally centred around a certain best guess (in contrast to multi-model distributed variables, that generally seem less plausible). It is worth to note that signalling in this model can be quantified by introducing a conditional *shift in the mean value* of the distribution. For example, State B might think that the audience cost b_S of state A for backing down is rather high, but through threatening, State A makes clear that it is lower — and the whole Gaussian distributed variable b_S (as estimated by state B) shifts. In this article, however, we focus on uncertainty, rather than the effect of shifting of the mean value resulting from a signal.

For clarity, we introduce a notation $C_x^{(y)}$, where C is one of the cost or payoff variables. The lower script x refers to the state (sender or target) to which the cost or payoff applies, e.g., b_S is the audience cost *for the sender*. The upper script y refers to from which perspective (sender or target) we are viewing the cost, e.g. $b_S^{(T)}$ is the audience cost for the sender, *from the perspective of the target*. Note that some variables are fully known by the party itself — $b_S^{(S)}$ is the sender's own audience cost as estimated by itself, hence we assume that she knows this, i.e., the uncertainty of this variable is zero. However, the uncertainty of other variables (like $b_S^{(T)}$) are dependent on how much the states know about each other and is modelled with Gaussian distributions, as described above.

We also assume that the costs of engaging in coercion $(-c_S, -c_T)$ are not fully known to the players — both their own payoffs and the opponents payoff. This allows us to account for the uncertainty related to a military intervention or economic coercion; factors like success, duration or intensity of the conflict are not fully known by both parties beforehand.

3.4 Dynamics under incomplete information

We now analyse the dynamics of the crisis bargaining model depicted in Figure 1 in the incomplete information framework, where each player faces uncertainty about the payoffs of the opposing state (and own payoff in case of engaging in coercion). In order to compare the pay-offs of each player at each decision node, we need to compute all the payoffs starting from phase t_4 backwards. While the complete information framework only has deterministic outcomes based on the distribution of payoffs, the incomplete information game is determined by a set of probabilities — p_1, p_2 and p_3 (see Figure 1). In contrast to existing literature, where these probabilities (if explicitly defined) are commonly given as a constant, we express these probabilities in terms of the payoff variables and their uncertainties, as we will illustrate in the remainder of this section. We use notation $N(\bar{x}, \sigma_x)$ for the normal distribution of variable x, with mean $\bar{x}$ and standard deviation σ_x. True values are denoted as just 'x', like in Figure 1, and in derivations we put $N(\bar{x}, 0) = x$ (although in theory, $\bar{x}$ can still be wrong). Analogous to payoffs, we write the probability $p_i^{(y)}$ as the probability seen from the perspective of state y.

Phase t_3

Similarly to the analysis of the complete information framework in Section 3.2 we begin by analysing the game tree from the end, starting with p_3, defined as the probability of the sender engaging in coercion (given that phase t_3 is reached), having pay-off $-c_S$. At the same time, the probability that the sender backs down is $1 - p_3$, achieving payoff $-1 - b_S$. As only the sender has an active choice in this phase, we only need to compare the payoffs of the sender to calculate both $p_3^{(S)}$ and $p_3^{(T)}$. Thus, the probability of engaging in conflict p_3 boils down to:

$$\begin{aligned} p_3^{(S)} &= P(-c_S^{(S)} > -1 - b_S^{(S)}) \\ &= P(-N(\bar{c_S}^{(S)}, \sigma_{c_S}^{(S)}) > -1 - b_S)) \\ &= P(N(0, \sigma_{c_S}^{(S)}) > -1 - b_S + \bar{c_S}^{(S)}) \end{aligned} \tag{1}$$

and similarly for the target (for whom both $c_S^{(T)}$ and $b_S^{(T)}$ are uncertain):

$$p_3^{(T)} = P(N(0, \sqrt{(\sigma_{b_S}^{(T)})^2 + (\sigma_{c_S}^{(T)})^2}) > -1 - \bar{b_S}^{(T)} + \bar{c}_S^{(T)}) \tag{2}$$

Phase t_2

Now that we have an expression for p_3, we work backwards towards phase t_2. We define p_2 (see Figure 1) as the probability that the target does not concede to a threat, given that a threat had been issued (analogously, $1 - p_2$ is the probability that the target concedes). To estimate p_2, we need to calculate the expected payoff of the target for not conceding (i.e., entering t_3), as estimated by the sender and by the target itself:

$$E_T^{(T)}(t_3) \quad = \quad p_3^{(T)} \cdot (-c_T^{(T)}) + (1 - p_3^{(T)}) \cdot (1 + b_T^{(T)}) \tag{3}$$

using the expression for $p_3^{(T)}$ from above. Furthermore, $b_T^{(T)} = b_T$ is known and $c_T^{(T)} = N(\bar{c_T}^{(T)}, \sigma_{c_T}^{(T)})$ is normal distributed, making $E_T^{(T)}$ a normal distributed variable. Similarly:

$$E_T^{(S)}(t_3) \quad = \quad p_3^{(S)} \cdot (-c_T^{(S)}) + (1 - p_3^{(S)}) \cdot (1 + b_T^{(S)}) \tag{4}$$

where $b_T^{(S)} = N(\bar{b_T}^{(S)}, \sigma_{b_T}^{(S)})$ and $c_T^{(S)} = N(\bar{c_T}^{(S)}, \sigma_{c_T}^{(S)})$ are normal distributed. Using the expressions for the expected payoff of the target in t_3, we can calculate p_2 as seen from both parties:

$$\begin{aligned} p_2^{(T)} &= P(E_T^{(T)}(t_3) > -a_T^{(T)}) \\ &= P(N(0, \sigma_{E_T^{(T)}(t_3)}) > -a_T^{(T)} - \bar{E}_T^{(T)}(t_3)) \\ &= P(N(0, \sigma_{c_T^{(T)}}) > -a_T^{(T)} - \bar{E}_T^{(T)}(t_3)) \end{aligned} \tag{5}$$

and:

$$\begin{aligned} p_2^{(S)} &= P(E_T^{(S)}(t_3) > -a_T^{(S)}) \\ &= P(N(0, \sqrt{(\sigma_{c_T}^{(S)})^2 + (\sigma_{b_T}^{(S)})^2 + (\sigma_{a_T}^{S})^2}) > -\bar{a}_T^{(S)} - \bar{E}_T^{(S)}(t_3)) \end{aligned} \tag{6}$$

Phase t_1

Analogously, we calculate the expected payoff of the sender for entering t_2 (by issuing a threat), viewed from both perspectives:

$$E_S^{(S)}(t_2) \quad = \quad p_2^{(S)} \cdot (E_S^{(S)}(t_3)) + (1 - p_2^{(S)}) \cdot (a_S^{(S)}) \tag{7}$$

and:

$$E_S^{(T)}(t_2) \quad = \quad p_2^{(T)} \cdot (E_S^{(T)}(t_3)) + (1 - p_2^{(T)}) \cdot (a_S^{(T)}) \tag{8}$$

where $E_S^{(S)}(t_3)$ and $E_S^{(T)}(t_3)$ are normally distributed. This leads to the following expression of p_1:

$$\begin{aligned} p_1^{(S)} &= P(E_S^{(S)}(t_2) > -1) \\ &= P(N(0, \sigma_{E_S^{(S)}(t_3)}) > -1 - \bar{E}_S^{(S)}(t_2)) \end{aligned} \tag{9}$$

and:

$$\begin{aligned} p_1^{(T)} &= P(E_S^{(T)}(t_2) > -1) \\ &= P(N(0, \sigma_{E_S^{(T)}(t_3)}) > -1 - \bar{E}_S^{(T)}(t_2)) \end{aligned} \tag{10}$$

3.5 Summary

We have analytically derived expressions for the probabilities p_1, p_2 and p_3. The only step towards getting numerical values for these probabilities is to assume the mean and standard deviation values for the parameters, and filling in these values into the above equations: the domestic audience costs (a_S, a_T, b_S and b_T), the payoffs in the case of conflict onset (already assumed to be -1 and 1), and economic costs resulting from a potential act of coercion (c_S and c_T). Of particular interest in the resulting probabilities are those from the active player's perspective; i.e., in phase t_1, $p_1^{(S)}$ is of interest because it determines the sender's decision to issue a threat (in contrast to $p_1^{(T)}$), in phase t_2, $p_2^{(T)}$ determines the target's decision to concede or not, and in phase t_3, $p_3^{(S)}$ determines the sender's final decision whether or not to engage in coercion.

Assuming average (true) values of the parameters equal to 1, we distinguish in this first exploration of the model three reference parameter sets: the 'incomplete' information case (all standard deviations equal to 1), a 'complete' information case (reference with standard deviations 0) and a 'no-information' case (reference with a very large standard deviation equal to 1,000) — the latter is added as a comparison[4]. Table I, summarises the most important probabilities in these reference sets. We can see that under complete information, the sender always threats, the target concedes in half of the cases, and the sender always follows up on the threat if it comes to that decision. In the other reference cases (i.e., when uncertainty is non-zero), the sender does not always issue a threat, the target concedes more often, and the sender does not always follows up on the threat. These differences between the complete and incomplete information cases display the role of information on its own, reflecting the importance of including uncertainty in a crisis bargaining model. Not only allows this framework to vary uncertainty, the model can also be tailored for different values of the payoffs. In the next section, we explore several parameter settings reflecting more real-world scenarios.

[4] In fact, we could have derived the outcome of the simulations in the 'no-information' column already analytically, because in a no-information case the probability of any choice of any actor in the decision tree is 50%.

Result	Symbol	Complete $\sigma \to 0$	Incomplete $\sigma = 1$	No information $\sigma \to \infty$
Action probabilities:				
Sender threats*	$p_1^{(S)}$	1	0.754	0.5
Target does not concede*	$p_2^{(T)}$	0.5	0.764	0.5
Sender engages in coercion*	$p_3^{(S)}$	1	0.841	0.5
End results:				
Status quo	$1 - p_1^{(S)}$	0	0.246	0.5
Concession target	$p_1^{(S)}(1 - p_2^{(T)})$	0.5	0.178	0.25
Backing down sender	$p_1^{(S)}p_2^{(T)}(1 - p_3^{(S)})$	0	0.091	0.125
Coercion onset	$p_1^{(S)}p_2^{(T)}p_3^{(S)}$	0.5	0.485	0.125

Table I. Summary of the probabilities in the reference parameter settings. In the last two columns we use settings with either very low or very high standard deviations. The asterisk indicates that the probability is conditional on that the particular phase has been reached.

4 Model results

In this section, we investigate the success probabilities of certain actions in the game tree for a range of parameter settings and derive empirically testable propositions. As noted above, if not otherwise stated, the distributions of uncertain variables (i.e., opposing player's payoffs and own payoff in t_4) are assumed to have a mean equal to the true value — often 1 —, and a standard deviations equal to 1. Analogous to Table I, we express the model results in probabilities — of which the analytical expressions are derived in the previous section. In particular, we map out the probability of a threat's success ($1 - p_2^{(T)}$) and the probability of engaging in coercion ($p_3^{(S)}$) (conditional on having reached these stages) for different payoffs settings. Note that although we use various parameter settings, all results displayed in this section are based on the analytically derived expressions of the previous section.[5]

4.1 Effectiveness of threats

As mentioned above (and visible in Fig. 1), $1 - p_2^{(T)}$ depicts the success probability of a threat, i.e., the probability that the target state concedes, given that a threat has been issued. Calculating this probability using the Eqn. 5 for varying values of the true value and standard deviation of the target's economic cost of coercion (c_T and σ_{c_T}, *ceteris paribus*) results in Fig. 2a. The results of the same procedure, but now varying the true value and standard deviation of the sender's audience costs of backing down (b_S and σ_{b_S}, *ceteris paribus*) are displayed in Fig. 2b. The horizontal axes in these panels show that an increase in the economic cost to the target [c_T, panel (a)] or an increase in the audience cost to the sender [b_S, panel (b)] causes the threat's success rate to rise, which is in line with the predictions in the crisis bargaining literature.

Furthermore, regarding uncertainty (the vertical axes), panel (a) shows that for relatively low

[5] There are no numerical simulations or Monte-Carlo methods used here. For details and user-friendly code, see our Github repository XYZ.

true economic cost to the target, more uncertainty in this cost leads to an increase in the probability of threat's success, while for relatively high economic cost to the target more uncertainty leads to a lower probability of threat's success. In panel (b), where we vary the (true value and uncertainty of the) sender's audience cost, an increase in the uncertainty results in threats being less likely to succeed. This is a novel prediction in the crisis bargaining literature, in which the effects of information and domestic audience cost are separated and distinguished.

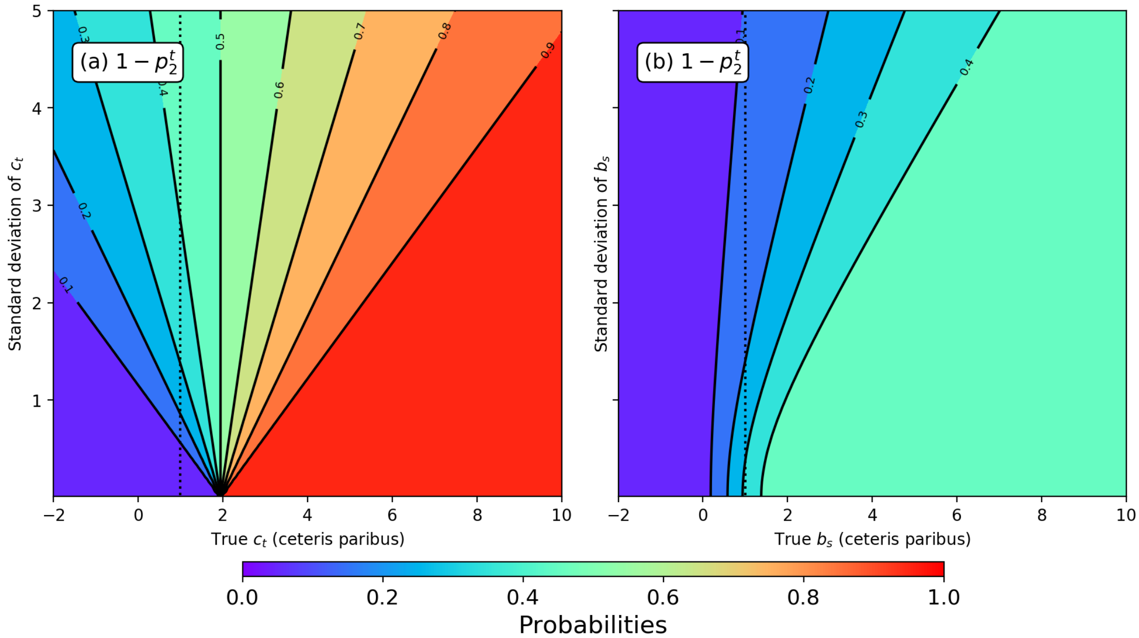

Figure 2. Probability of threat success $(1-p_2^t)$ in colours, given that a threat has been issued. Panel (a) shows the effect of varying the true economic coercion costs to the target (horizontal) and standard deviation of these costs (vertical). Panel (b) shows the effect of varying the true sender's audience cost of an empty threat (horizontal) and the standard deviation of this value (vertical).

Based on the above analysis, we can derive the following three propositions:

Proposition 1. In a conflict between states, increase in the economic cost to a target increases the effectiveness of a threat.

Proposition 2. In a conflict between states, increase in the domestic audience cost to a sender increases the effectiveness of a threat.

Proposition 3a. In a conflict between states, increase in the uncertainty about the economic cost to a target decreases (increases) the effectiveness of threats, for relatively high (low) true value of the economic cost.

Proposition 3b. In a conflict between states, increase in the uncertainty about the domestic audience cost to a sender decreases the effectiveness of threats.

The first two propositions that we offer relate to hypotheses that have already been established in the literature on crisis bargaining. Our first proposition reflects the coercive hypothesis — an expectation that with an increase of the cost of economic sanctions or war, relative to the size of the sender's and target's economy, coercion is more likely to end, and succeed, at the threat stage (Whang et al., 2013; Katagiri and Min, 2019). Our second proposition reflects the public commitment hypothesis — an expectation that states with a high domestic audience cost (e.g. democracies) are more likely to succeed at a threat stage (Fearon, 1994; Schultz, 2001, 1999; Katagiri and Min, 2019).

The third proposition is a novel contribution to the crisis bargaining literature and concerns the impact of information. It shows that uncertainty is a separate determinant next to domestic audience and economic cost in driving success of threats of coercion. This results is also in contrast with part of the crisis bargaining theory, where information and domestic audience cost were presented as a single determinant (Schultz, 1999, 2001; Whang et al., 2013). Here, we disentangle these mechanisms and show that they can work in parallel.

4.2 Engagement in coercion

Next, we consider the behaviour of states in respect to engagement in coercion, based on our crisis bargaining model. In particular, the probability that the sender decides to engage in coercion, when the game had already reached t_3. Next to the effectiveness of threats, this is another focal point in the theory development on international conflict. Figure 3 shows the probability of a sender engaging in coercion ($p_3^{(S)}$, given that a threat has been resisted) in colours. Analogous to Figure 2, panel (a) displays this probability for true values (horizontal axis) and standard deviation (vertical axis) of the sender's economic costs (in contrast to the *target's* costs in Figure 2). Panel (b) shows this probability for varying true values (horizontal axis) and standard deviation (vertical axis) of the sender's audience cost of backing down.

After an inspection of Figure 3, we conclude that a higher economic cost to the sender (as we would intuitively predict) reduces the probability of sender's engagement in a coercion. What is more, based on panel (b) in Figure 3, we observe that a higher audience cost to the sender increase the probability of coercion onset, what also follows an intuitive prediction. Interestingly, the role of uncertainty is again highlighted in these panels — high probabilities of engaging in coercion (for low economic costs or high audience costs) are decreased with higher uncertainty, and vice versa.

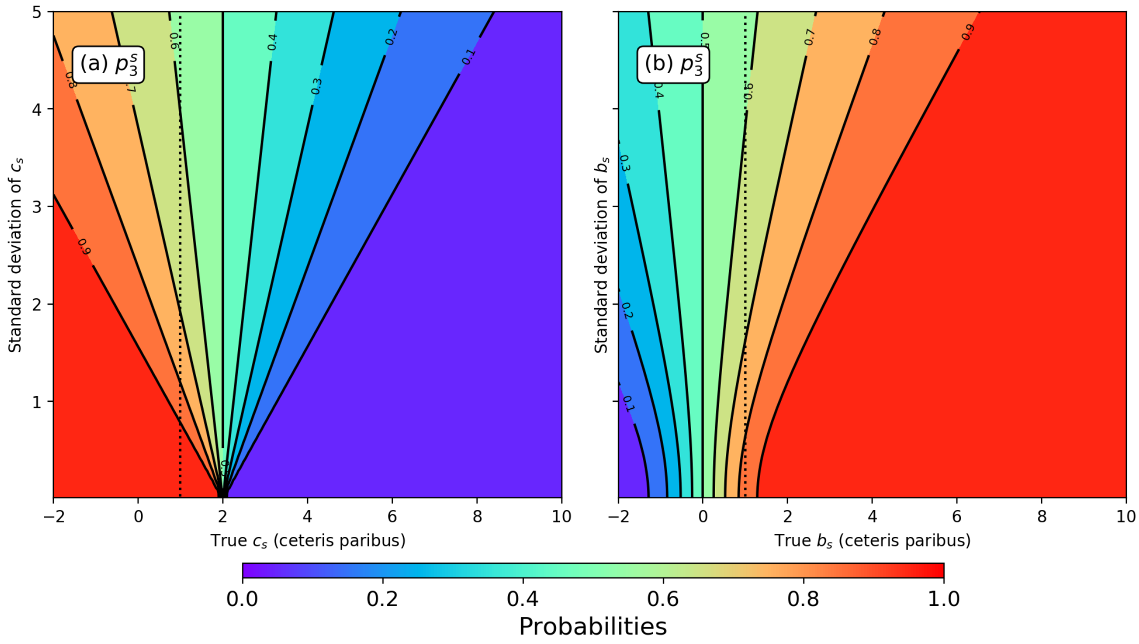

Figure 3. Probability of engaging in coercion (p_3^s) in colours, given that a target did not concede to a threat. Axes as in Fig. 2.

Based on our model's prediction, we have generated three propositions on coercion onset:

Proposition 4. In a conflict between states, increase in the economic cost to a sender decreases the probability of a coercion onset.

Proposition 5. In a conflict between states, increase in the audience cost to a sender increases the probability of a coercion onset.

Proposition 6a. In a conflict between states, increase in uncertainty decreases the probability of coercion onset, for relatively high true value of both economic and audience cost to the sender.

Proposition 6b. In a conflict between states, increase in uncertainty increases the probability of coercion onset, for relatively low true value of both economic and audience cost to the sender.

In the next two sections, we explore our crisis bargaining model by applying it to two conflict scenarios: (I) a democratic sender threatens an autocratic target and (II) both sender and target experience a high audience cost. Furthermore, we offer a programme that allows scholars to themselves change our assumptions and generate scenarios and respective predictions.[6]

4.3 Scenario I - A democratic sender threatens an autocratic target

In the first scenario, the target's domestic audience cost is lower than the cost of engagement in coercion ($a_T < c_T$) and the sender's domestic audience cost is higher than the cost of an engagement in coercion ($a_S > c_S$). This scenario mimics a typical conflict situation between a democratic

[6]This programme is available via a Github repository.

sender and an autocratic target; scholars indicate that authoritarian regimes can successfully shield themselves from popular discontent (Allen, 2008) (i.e., low domestic audience costs for authoritarian regimes), that voters in democratic regimes punish political leaders for issuing empty threats (Kertzer and Brutger, 2016) (i.e., high domestic audience costs for democracies), and that democracies, on average, boast more complex and resilient economies than authoritarian regimes (Farrell and Newman, 2019) (i.e., economic costs of coercion are relatively lower for democracies). This theoretical set-up is particularly relevant to instances of economic sanctions and non-proliferation, for example US sanctions against Iran or North Korea.

To analyse this scenario in the framework of our crisis bargaining model, we set $a_T = b_T = 0.5, c_T = 1.5, a_S = b_S = 1.5$ and $c_S = 0.5$. Table II shows the resulting probabilities for three cases of the standard deviations as in Table I. In a complete information case (second-last column), the game always ends in the target conceding, as it knows that the sender will engage in coercion. The incomplete information case column (middle column) shows that the probability of the target not conceding becomes greater than 0. Yet, the probability of the sender backing down is still marginal, pointing towards the dominating effect of domestic audience costs at the final decision stage. The audience costs-driven determination of the sender to engage in coercion in this scenario setting may reflect the determination of the Trump administration in sanctioning of Iran over nuclear proliferation concerns.

Table II. Summary of probabilities in the parameter settings of Scenario I. In the last two columns we use settings with either very low or very high standard deviations. Standard deviations are only non-zero (in any of the cases), when considering pay-offs of the opposing player or own conflict costs.

Result	Symbol	Incomplete $\sigma = 1$	Complete $\sigma \to 0$	No information $\sigma \to \infty$
Action probabilities:				
Sender threats*	$p_1^{(S)}$	0.971	1	0.5
Target does not concede*	$p_2^{(T)}$	0.222	0	0.5
Sender engages in coercion*	$p_3^{(S)}$	0.977	1	0.5
End results:				
Status quo	$1 - p_1^{(S)}$	0.029	0	0.5
Concession target	$p_1^{(S)}(1 - p_2^{(T)})$	0.755	1	0.25
Backing down sender	$p_1^{(S)} p_2^{(T)}(1 - p_3^{(S)})$	0.005	0	0.125
Coercion onset	$p_1^{(S)} p_2^{(T)} p_3^{(S)}$	0.211	0	0.125

* given that the particular phase has been reached.

4.4 Scenario II - High audience costs for sender and target

A second scenario considers the case of both the sender and the target state experiencing higher domestic audience costs than economic coercion costs. This theoretical set-up reflects cases in which democratic senders and target states that are newly formed (authoritarian) regimes which have not yet consolidated power and do not want to appear weak to domestic competitors (Spaniel and Smith, 2015).[7] This scenario may reflect the US-Venezuela conflict, where the regime of Maduro is in a power-transition period since year 2013, after the death of Hugo Chavez, and faces a number of domestic competitors.

To analyse this scenario in our model we set $a_T = b_T = a_S = b_S = 1.5$ and $c_T = c_S = 0.5$ (see Figure 1 for reference); we report the resulting probabilities in Table III. In the complete information case (second-last column), the parties eventually always engage in coercion, and therefore, threats are never successful. However, in an incomplete information game (middle column), we observe that the effectiveness of threats increases (11% chance of target's concession). Nevertheless, we observe that both the target is not very likely to concede and the sender is very likely to engage in a conflict. This may serve as a reference point to the on-going US-Venezuela conflict, where the Maduro regime rejected humanitarian aid to Venezuelans in order to not appear weak and there appears little sight of a solution. This example points to two potential proxies of uncertainty. One could be duration of a regime; actors cannot rely on past experience (own and of other states) when dealing with a relatively young regime. This may add to the uncertainty about the costs and benefits that the regime faces. Second, diplomatic ties may reflect the level of uncertainty. States with frequent diplomatic exchanges may be more likely to precisely estimate the costs and benefits of the opponent in a crisis bargaining situation.

Table III. Summary of probabilities in the parameter settings of Scenario II. In the last two columns we use settings with either very low or very high standard deviations.

Result	Symbol	Incomplete $\sigma = 1$	Complete $\sigma \to 0$	No information $\sigma \to \infty$
Action probabilities:				
Sender threats*	$p_1^{(S)}$	0.842	1	0.5
Target does not concede*	$p_2^{(T)}$	0.892	1	0.5
Sender engages in coercion*	$p_3^{(S)}$	0.977	1	0.5
End results:				
Status quo	$1 - p_1^{(S)}$	0.158	0	0.5
Concession target	$p_1^{(S)}(1 - p_2^{(T)})$	0.091	0	0.25
Backing down sender	$p_1^{(S)}p_2^{(T)}(1 - p_3^{(S)})$	0.017	0	0.125
Coercion onset	$p_1^{(S)}p_2^{(T)}p_3^{(S)}$	0.734	1	0.125

* given that the particular phase has been reached.

5 Conclusions

In this article, we put forward an extended game theory model of crisis bargaining for international conflict which goes beyond the current literature in the following ways: (a) we introduce uncertainty

[7] It may also be applied to a conflict between two democracies.

on both domestic audience and economic cost, (b) we formalise what uncertainty means and quantify its distribution, we (c) show that uncertainty and costs are separate mechanisms and finally, (d) we offer a distinction between uncertainty and signalling. This extended framework is consistent with the main predictions of the existing models but at the same time implies novel and empirically testable propositions relating to the response of a target state to a threat and the response of a sender to a target that did not concede to a threat.

Consistent with earlier work, our model predicts that both an increase in the economic cost to a target and an increase in the domestic audience cost to a sender increases the effectiveness of a threat. In addition, our model predicts that an increase in the uncertainty about the economic cost to a target decreases the effectiveness of threats, if the actual value of the economic cost is relatively high. We also observe that an increase in the uncertainty about the domestic audience cost to a sender decreases the effectiveness of threats. When uncertainty increases, the target is more likely to decide to resist a threat of coercion that would have been successful under complete information, or with less uncertainty. With respect to engagement in coercion, our model predicts that an increase in the economic cost to a sender decreases the probability of a coercion onset, while an increase in the audience cost (to a sender) increases the probability of engagement in coercion. Finally, we observe that uncertainty has an equalising effect on coercion onset and decreases (increases) the probability of coercion, for relatively high (low) actual values of both economic and audience cost to the sender.

More and more empirical data that offers information on conflicts that did not escalate beyond a threat is available. For example, the TIES data set (Morgan et al., 2014) includes cases of economic sanctions that terminated at the threat stage (failures and successes). Our general framework offers guidance for research into the conditions that systematically affect effectiveness of threats and the prospects of coercion onset conditional on a failed threat. A large body of scholarly work — for example findings of Whang et al. (2013); Whang and Kim (2015); Lacy and Niou (2004) on effectiveness of threats and of Peksen (2019); Hafner-Burton and Montgomery (2008); Schultz (1999); Spaniel and Smith (2015) on onset of coercion — may be reconsidered in the light of this article. A set of hypotheses based on the existing crisis bargaining framework would not allow empirical research to take full advantage of, for example, the TIES data set (Morgan et al., 2014) and observations of conflict in international relations that terminated at the threat stage. In addition, our model is readily accessible to researchers and can easily be adjusted for desired values and generate visualisation of predictions.

The model and associated code can be further developed by including signalling to account for the role of information in even more detail. Currently, we only study the effect of change in variance around a mean value — our measure of uncertainty. However, a shift in the mean value of an estimate would, due to a change from status quo to a threat, could potentially reflect what scholars refer to in the literature as signalling.

In summary, this article offers a synthesis and an advancement of scholarship on crisis bargaining. We offer a single framework for international conflict under complete and incomplete information, in the form of a game theory model that allows for variation in uncertainty. Crucially, our model makes the novel prediction that the probabilities of a threat's success and coercion onset depend both on the economic and domestic audience costs, as well as on the uncertainties about these costs. Finally, the generality of our model allows for broad application in empirical research and offers scope for further investigation of many subjects related to information, such as signalling or bluffing.

References

Allen, S. H. (2008). The Domestic Political Costs of Economic Sanctions. *Journal of Conflict Resolution 25*(6), 916–944.

Arena, P. (2013). Costly Signaling, Resolve, and Martial Effectiveness.

Bas, M. A. and R. Schub (2018). The Theoretical and Empirical Approaches to Uncertainty and Conflict in International Relations. In W. R. Thompson (Ed.), *Oxford Research Encyclopedia of Politics*, pp. 177–194. Oxford: Oxford University Press.

Blainey, G. (1988). *The Causes of War*. London: Macmillan.

Drezner, D. (2003). The Hidden Hand of Economic Coercion. *International Organization 57*(03), 643–659.

Farrell, H. and A. L. Newman (2019). Weaponized Interdependence. *International Security 44*(1), 42–79.

Fearon, J. (1994). Domestic Political Audiences and the Escalation of International Disputes. *American Political Science Review 88*(03), 577–592.

Fearon, J. (1997). Signaling foreign policy interests: Tying hands versus sinking costs. *Journal of Conflict Resolution 41*(1), 68–90.

Fearon, J. D. (1995). Rationalist Explanations for War. *International Organization 49*(3), 379–414.

Gartzke, E. (2007). The Capitalist Peace. *American Journal of Political Science 51*(1), 166–191.

Gartzke, E. A., S. Carcelli, J. A. Gannon, and J. J. Zhang (2017). Signaling in Foreign Policy. *Oxford Research Encyclopedia of Politics*, 1–30.

Goldstein, J. S. (2011). *Winning the War on War: The Decline of Armed Confict Worldwide*. New York: Dutton.

Hafner-Burton, E. M. and A. H. Montgomery (2008). Power or Plenty. How Do International Trade Institutions Affect Economic Sanctions? *Journal of Conflict Resolution 52*(2), 213–242.

Johnson, D. D. (2004). *Overconfidence and War.* Cambridge, MA: Harvard University Press.

Katagiri, A. and E. Min (2019). The Credibility of Public and Private Signals: A Document-Based Approach. *American Political Science Review 113*(1), 156–172.

Kertzer, J. D. and R. Brutger (2016). Decomposing Audience Costs: Bringing the Audience Back into Audience Cost Theory. *American Journal of Political Science 60*(1), 234–249.

Kydd, A. (2003). Which Side Are You On? Bias, Credibility, and Mediation. *American Journal of Political Science 47*(4), 597–611.

Lacy, D. and E. M. Niou (2004). A Theory of Economic Sanctions and Issue Linkage: The Roles of Preferences, Information, and Threats. *Journal of Politics 66*(1), 25–42.

Morgan, T. C., N. Bapat, and Y. Kobayashi (2014). Threat and imposition of economic sanctions 1945–2005: Updating the TIES dataset. *Conflict Management and Peace Science 31*(5), 541–558.

Morrow, J. D. (1989). Capabilities, Uncertainty, and Resolve: A Limited Information Model of Crisis Bargaining. *American Journal of Political Science 33*(4), 941–972.

Neuenkirch, M. and F. Neumeier (2016). The impact of US sanctions on poverty. *Journal of Development Economics 121*, 110–119.

Partell, P. J. and G. Palmer (1999). Audience costs and interstate crises: An empirical assessment. *International Studies Quarterly 43*(2), 389–405.

Peksen, D. (2019). When Do Imposed Economic Sanctions Work? A Critical Review of the Sanctions Effectiveness Literature. *Defence and Peace Economics 30*(6), 635–647.

Pinker, S. (2011). *The Better Angels of Our Nature.* New York: Viking.

Pond, A. (2017). Economic Sanctions and Demand for Protection. *Journal of Conflict Resolution 61*(5), 1073–1094.

Powell, R. (1988). Nuclear Brinkmanship with Two-sided Incomplete Information. *American Political Science Review 82*(1), 155–178.

Schultz, K. (1999). Do Domestic Institutions Constrain or Inform? *International Organization 53*(2), 233–266.

Schultz, K. (2001). *Democracy and Coercive Diplomacy.* Cambridge: Cambridge University Press.

Signorino, C. S. (1999). Strategic Interaction and the Statistical Analysis of International Conflict. *American Political Science Review 93*(02), 279–297.

Slantchev, B. L. and A. Tarar (2011). Mutual Optimism as a Rationalist Explanation for War. *American Journal of Political Science 55*(1), 135–148.

Smith, A. and A. C. Stam (2004). Bargaining and the Nature of War. *Journal of Conflict Resolution 48*(6), 783–813.

Snyder, J. and E. D. Borghard (2011). The cost of empty threats: A penny, not a pound. *American Political Science Review 105*(3), 437–456.

Spaniel, W. and B. C. Smith (2015). Sanctions, Uncertainty, and Leader Tenure. *International Studies Quarterly 59*(4), 735–749.

Trachtenberg, M. (2012). Audience costs: An historical analysis. *Security Studies 21*(1), 3–42.

Van Evera, S. (1999). *Causes of War: Power and the Roots of Conflict.* Ithaca, NY: Cornell University Press.

Wagner, H. R. (1994). Peace, War, and the Balance of Power. *American Political Science Review 88*(3), 593–607.

Whang, T. and H. J. Kim (2015). International Signaling and Economic Sanctions. *International Interactions 41*(3), 427–452.

Whang, T., E. V. Mclean, and D. W. Kuberski (2013). Coercion, Information, and the Success of Sanction Threats. *American Journal of Political Science 57*(1), 65–81.

Wolford, S., D. Reiter, and C. J. Carrubba (2011). Information, Commitment, and War. *Journal of Conflict Resolution 55*(4), 556–579.