235 Years since the Electoral College: A Probabilistic Consideration of Voting Power

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Abstract

This paper analyzes the extent to which the electoral college system in United States presidential elections disenfranchises and diminishes the voting power of particular geographic populations. In the study, eligible voting bodies are measured by their ability to impact the outcome of the election. We used Monte Carlo simulations, which generate randomly simulated elections where each state has an equal chance of voting for or against the winning president. We then calculated which states were able to *swing* the outcome of the vote from winning to losing to find Banzhaf's power index. To provide a more comprehensive understanding of individuals' power, the same calculations were performed on various populations. The study found significant disparities in voting power between different states, and between their respective residents. With consideration to the recent electoral inversions, we hope to add to the conversation surrounding the electoral college using new data and analysis.

Keywords — Banzhaf Index, Power Index, Electoral College, Voting Power, Voting Disparity, Electoral Inversions, Geospatial Power

1 Introduction

"A man without a vote is a man without protection."

–Lyndon B. Johnson (36th President of the United States)

The electoral college saw renewed attention in the past few decades, as questions regarding its utility and effective biases have entered the mainstream in political, legal, and popular debate. Arguments stemming from positions of historical reverence, constitutional sovereignty, protection of minority voices, the inherent disproportionately of a weighted system, and the apparent allowance of faithless electors arose from their seemingly restless slumber in response to recent instances of electoral inversion [7]. Two of the past five presidents to hold office, Bush in 2000 and Trump in 2016, failed to win the popular vote, yet still won office. The previous and only three presidents to likewise attain this anomaly are separated by spans of hundreds of years with abnormal conditions, rather than purely intrinsic structural qualities, propelling multiple into office [1]. This outwardly low rate of anomalies must not be discounted as an inevitable necessity of a proud-standing, strong institution safeguarding the rights of the people and the integrity of the nation. It is indicative of a far deeper fear spreading throughout American politics: the likelihood that more presidents can win the necessary electors while losing the popular vote [12]. It is still in dispute whether the electoral college is an overall net positive to society or a derelict remnant of earlier times holding back our ability to fully embrace contemporary democracy and enter into the modern era.

Power is often attributed to the ability to make change, yet when it comes to voting, it is possible that each voter may not have the same amount of power [3]. Each rationale challenging or defending the electoral college is concerned in some manner with how it shapes and allows for the exercise of power. An appeal to the system's historical longevity may acknowledge that underneath it there has been a relatively consistent balance of power, resulting in stability and peace as the political pendulum swings back and forth, allowing rural areas to be heard [13]; a more modern criticism of the system may identify disparities in voting proportion, as smaller states receive a greater share of electors despite their smaller populations, and dismiss the system as oppressive and unjust, skewing the will of the people and favoring rural and isolated interests. Each argument has its merits and posits a reasonable, if limited, understanding of the situation, yet neither is able to fully encapsulate the scope of power, nor accurately assess the extent to which it is distributed or allowed to be exerted.

Quantifying power first requires a clear, standardized definition, one such popularly applied to voting bodies is the extent to which a voter is able to influence elections and their outcomes. To measure power in its actuality would be an extraordinarily complex endeavor, necessitating the consideration of countless factors, ensuring there will always be disputes as to the balance of it within a body. The Banzhaf model, commonly used to study voting power, operates under the assumption of limited bias, focusing on the hypothetical [5]. Our study, however, attempts to explore and expand upon different factors of power by referring to probabilistic models like Banzhaf to measure the *a priori* power awarded by the electoral college.

2 Background

The electoral college was founded by the Framers of the Constitution as a measure of federalism, specifically aiming to give states a say in who runs the national government [13]. It is up to the states' government to determine the process of how electors are awarded. Currently, 48 states operate under a winner-take-all system, with all of the state's electors being awarded to the candidate that won the state's popular vote. During Election Day, citizens are actually voting to allot electors to a certain candidate. These electors then vote and select the president. Depending on the state, electors are not bound to represent the state's popular vote, those that break from the popular vote are called *faithless electors*. There have been seven faithless electors of the presidency, all in the 1900s, with another elector casting a blank ballot in 2000 [14]. No faithless elector has changed the outcome of the presidency, but their existence has drawn scrutiny in the 17 states without faithless elector laws [20]. States that do not follow the winner-take-all are Maine and Nebraska, opting for a district apportionment system.

Criticisms of the district apportionment system revolve around the accusation of gerrymandering (the process of redistricting to favor one party). The Supreme Court case *Baker v. Carr* outlawed gerrymandering as a violation of the Fourteenth Amendment and established that the system of apportionment could be judiciously reviewed. Later in *Reynolds v. Simms*, the Supreme Court noted that the Fourteenth Amendment requires apportionment to be based on equal populations. Further, the argument is that the district apportionment system will cause gerrymandering to occur more frequently as state legislatures redraw districts to favor one party [2]. While the Supreme Court has outlawed gerrymandering, the Supreme Court has refused to comment on how states appoint their electors, reaffirming states' rights to appoint their electors as they see fit in *Williams v. Va. State Brd. of Elections* [25]. In the 2019 case *Rucho v. Common Cause*, the Supreme Court ruled that gerrymandering is a legislative problem and out of the judicial realm of the courts [16]. This further establishes the electoral college as a pillar of federalism in which apportionment is based entirely upon state decisions.

Though the apportionment of electors is a state decision, there have been few recent studies across state lines revolving around the amount of power attributed to different states' residents. Many argue that the smaller states receive too much power, as states receive an automatic three votes. The other side believes that the winner-take-all system favors the larger states. To explore which side is correct using mathematical reasoning, power indices are created to represent the probabilities of winning elections. The first paper was the Shapley-Shubik study which revealed that power is not proportional to its size when studying the Congressional elections [18]. Applying a probabilistic model similar to Shapley-Shubik, Banzhaf devised a way to calculate the power of each individual citizen in the electoral college [4]. The predominant difference between the Shapley-Shubik index and the Banhzaf index is that the order in which calculations are performed matter when calculating the Shapley-Shubik index. Criticisms of the indices primarily revolve on its inability to calculate *a posteriori power* and focuses on the hypothetical [24]. For the electoral college, where the order in which states vote do not matter, the Banzhaf index is most commonly used, as it provides the power of an individual citizen as well as the power of an entire state. This study expands upon the Banzhaf study, calculating power for the individual citizen, the individual registered voter, and the individual voter.

3 Methodology

Considering improvements in technology since Banzhaf's original paper [5] and the renewed interest in the electoral college, applying the Banzhaf Power Index methods to current circumstances sheds light on the role the electoral college plays in maintaining a fair election. With computer programs, more elections can be simulated, offering a clearer picture of the electoral college. Research happened in two parts: designing a computer program and running several Monte Carlo simulations as well as final calculations made with the collected data.

For the initial part, the program was designed to simulate several elections in which each time a state has an impact on the outcome of the election is recorded. As explained in the Mathematics subsection, every possible outcome of the electoral college amounts to approximately $1.12E + 15$, so one must use a smaller size known as a Monte Carlo simulation. It is necessary to use the aid of a computer to speed up computations that represent a portion of the electoral college. This information and the Normalized Banzhaf Power Index is displayed for each state. For this study, over 15 simulations containing 10 million elections each were ran, amounting to 150 million elections.

Taking the recorded amount of swings were recorded as well as the Normalized Banzhaf Power Index, the operations discussed in the Mathematics subsection were performed. The results are the Normalized Banzhaf Power Index, the Absolute Banzhaf Power Index, and the Banzhaf Relative Strength for three populations. To confirm the computer data labeled as Normalized Banzhaf Power Index with the correct definition as dictated in literature, the formula 2 was performed with only the recorded amount of swings. It was found that the Normalized Banzhaf Power Index by computer calculation and by hand were the same. For the Banzhaf Relative Strength calculations, US Census Bureau tables were used to give information on three populations including standard resident population (after the 2020 Census) [23], voter registration, and voter turnout (pertaining to the November 2020 election) [22]. The average of the 15 trials for the *swings,* Normalized Banzhaf Power Index, and Absolute Banzhaf Power Index are given in Table 1. The final calculations for the Banzhaf Relative Strength are recorded in Table 5 using the average Absolute Banzhaf Power Index as described in the Mathematics subsection.

3.1 Mathematics

Reviewing the previously established mathematics, there are two commonly used definitions of the Absolute Banzhaf Power Index. The Absolute Banzhaf Power Index is said to be the division of the number of winning coalitions by the number of all possible coalitions. In the case of the United States, we say 2^{n-1} is 2^{50} which is approximately $1.12E + 15$ coalitions.

$$\beta'_a[W] = \frac{\eta_a[W]}{2^{n-1}} \tag{1}$$

The Normalized Banzhaf Power Index is said to be the division of the number of winning coalitions (*swings*) by the number of total winning coalitions (*total swings*) across the bodies analyzed.

$$\beta_a[W] = \frac{\eta_a[W]}{\Sigma_{x \in N}\ \eta_x[W]} \tag{2}$$

It is important to note that Owens in his paper [15] exploring multilinear extensions of the Shapley-Shubik and Banzhaf values, his version of the total Banzhaf value Ψ_i is the same as the Normalized Banzhaf Power Index or Ψ_i=$\beta_a[W]$.

The two previously mentioned indices involve only analysis between the states and their electoral votes. However, they do not offer any information about each individual's effect on the election. Banzhaf in his paper [4] proposes the term Banzhaf Relative Strength, as a measure of one citizen's influence compared to another citizen's influence, where the latter resides in the weakest state.

$$\beta_i = \beta'_x \div \beta'_\omega \tag{3}$$

Before calculating Banzhaf Relative Strength, we calculate the influence of one's person vote on the election for each state. We define this as the Relative to Population value. To calculate this influence, we can multiply the probability that one person will change their state's outcome by the probability that the state is critical in a coalition.

$$\beta'_x = \rho_x * \beta'_a[W] \tag{4}$$

The probability that one person will change their state's outcome can be calculate is equivalent to the square root of 2 divided by the product of pi and the population of the state.

$$\rho_x = \sqrt{\frac{2}{\pi * n_x}} \tag{5}$$

Commonly called the Penrose square root law, this rule can be derived using Stirling's approximation, which concerns the expansion of factorials [8, 11]. As the population increases, yet only two options remain, it can be observed that the probability seems to approach a limit. This trend is determined by pi, the population, and two.

We can combine both the Absolute Banzhaf Power Index (1) with the probability that one person will change their state's outcome (5) to create an alternative equation for the Relative to Population value.

$$\beta'_x = \rho_x * \beta'_a[W] \tag{6}$$

$$= \sqrt{\frac{2}{\pi * n_x}} * \frac{\eta_a[W]}{2^{n-1}} \tag{7}$$

We sort each of the Relative to Population values, and we select the lowest term and call it β'_ω. This is the necessary information to calculate the Banzhaf Relative Strength (3) for citizens from all 51 voting blocs.

In previous literature, another ratio termed the Banzhaf Ratio ν_i represents the Absolute Banzhaf Power Index of one state to the Absolute Banzhaf Power Index of the weakest state ($\beta'_\omega[W]$).

$$\nu_i = \frac{\beta'_a[W]}{\beta'_\omega[W]} \tag{8}$$

The utilization of a computer and spreadsheet technology allows one to easily determine the weakest state. In turn, this expands the number of populations the Banzhaf Relative Strength calculation (3) can be applied to. The chosen population becomes n_x when calculating the probability that one person will change their state's outcome (5). The three populations chosen illustrate how people's power change when the redistricting is based on a different population category instead of the standard residents population.

Due to the newness of the Census data and the large amount of possibilities to consider, there is no accepted or known actual value, so the accuracy of the results is unknown. However, we can calculate the amount of error estimated in the data. Monte Carlo error can be found by multiplying the sample variance by three and dividing by the root of the number of trials N_t [11].

$$\varepsilon = 3 \times s^2 \div \sqrt{N_t} \tag{9}$$

Variance is the square of the difference of each data point from the mean $\bar{x}$.

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2} \tag{10}$$

In this case, each variance of each individual state over the 15 trials was averaged for the Absolute and the Normalized Banzhaf Power Indices. Performing this gives a comprehensive error estimate for all 51 areas analyzed.

3.2 Computer Programming

The program that computed the approximated Banzhaf indexes was written in the Java programming language. The program is split into four classes: *Banzhaf, Parse, State*, and *PowerAlgo*. The Banzhaf class is the main class and calls functions from those other classes and uses them together to achieve the program's goal.

The main function begins by using the Parse class to read *states.txt,* a text file, and returns an ArrayList that is easier for the program to process. The text file contains the names and electoral votes of each state, meaning that the program is completely configurable for any real or hypothetical election by changing the data in the text file.

The states ArrayList created by the Parse class is an ArrayList of State objects. State objects are defined by the State class, and include information on the state's name, votes, power (the total number of election swings), and power index.

The states ArrayList is then passed to the PowerAlgo class, which does the majority of the math of the program. As is done when calculating a Banzhaf power index by hand, coalitions are made by the program, and the states that are able to change the election on their own by switching have 1 added to their power. After a large number of coalitions are generated and states' powers are adjusted, the list is returned to the Banzhaf class. As previously mentioned, the large number of states in the United States and the extreme amount of possible elections with a country so large, not all possible elections are simulated. Instead, a sufficiently large number of elections are simulated to very closely approximate the true Banzhaf power indices. This reduces computing time, and multiple iterations are used to provide more precise data.

Finally, the program adds the total number of times each state had the power to change an election, and calculates the power index of each state by dividing states' powers by the total power of all the states. The code then printed each state out along with its respective Banzhaf power index. These values were used for the data contained in the study and are attached in the supplementary materials.

3.3 Methodological Limitations and Assumptions

The Banzhaf metrics are a measure of *a priori* power and is independent of factors that influence voting. We operated under the assumption that each state and each individual voter would operate individually. This assumption meant a fifty-fifty chance of voting each way. While it can be said that each voter does not normally consider each candidate equally and each state usually belongs to a partisan voting bloc [10], it is beyond the scope of the article to discover how a voter's power may change based on partisan views. The opposite of theoretical (*a priori*) voting is *a posteriori* voting power, often termed as *actual* voting power. It is also necessary to make the distinction that the Banzhaf metrics are a normalized index of I-power. Felsenthal, the accomplished writer on voting power models, says I-power is termed as a model where people vote for policy, whereas P-power is described as a model where people vote due to office-seeking [9]. It should be noted that in an ideal simulation, states with the same amount of electoral votes would have the same Absolute Banzhaf Power Index and same values for the Normalized Banzhaf Power Index. In addition, Nebraska and Maine, which use electoral districts, have their total electors grouped into one. Though some districts have split on the presidential election, giving votes to two different candidates, splitting Nebraska and Maine would add to the computational complexity of the calculations. It should be noted that the only times Nebraska and Maine split their electoral votes to two different candidates were in 2012 and 2020 (Nebraska) as well as 2016 and 2020 (Maine) [6], [17]. As the splits are recent, it may indicate that the winner-take-all system is becoming more outdated. While these assumptions may limit one's perception that the Banzhaf metrics are unreliable, on the contrary, it offers several opportunities for further research and an examination of how the electoral college system has many factors such as Straffin's formula of political bias [19]. The Banzhaf metrics merely represent how much control one person has in the outcome of an indirect election model such as the electoral college.

4 Results

The Banzhaf Relative Strength metrics show that residents of more populous states tend to have more votes than someone from a less populated state. When considering the standard resident population used in the redistricting for the 2024 and 2028 election, Californian residents have 3.4928 votes compared to Idaho's 1 vote. Using voter registration from the 2020 presidential election, Californian residents improve their ability to affect the elected president to 3.6212 and Idaho residents are left with a 1 vote weight. Considering voter turnout from the 2020 presidential election, Californian residents have a slight decrease from the voter registration Relative Strength, but still an increase from the standard population count at 3.6178. Again, Idaho residents are left with 1 vote. It can be seen that the two ends of the spectrum remain the same regardless of the population analyzed, but there are significant differences for other states. One example is DC, which is negatively impacted by the different populations. DC residents have a smaller impact on the presidential election, moving down 12 rankings when considering voter turnout and down 14 when considering voter registration. One state that benefited from considering alternative populations is Arkansas, moving up 12 rankings when considering voter turnout and up 13 when considering voter registration. The overall ranks for the Banzhaf metrics can be seen in Table 4.

When looking at the Banzhaf Ratio ν_i (refer back to (8)), DC residents have the weakest affect compared to the rest of the US. In addition, looking at voter registration and voter turnout hurts DC residents' affects on the election, despite DC having a larger voting turnout percentage than California historically [21]. The other Banzhaf metrics which do not involve citizen population Absolute Banzhaf Power Index 1, the Normalized Banzhaf Power Index 2, and the Banzhaf Ratio 8 show that the states with the most electoral votes tend to have the highest impact on the presidential election, while states with a smaller amount of electoral votes tend to have a smaller impact on the presidential election.

The amount of error is less than one-hundred thousandth of a percent for the Normalized Banzhaf Power Index and less than one-ten thousandth of a percent for the Absolute Banzhaf Power Index as seen in Table 2. The error is minimal, demonstrating the accuracy of our code's Monte Carlo simulations. This minimal error is extended to the calculations applied post-simulations.

5 Analysis

The results of this paper confirm Banzhaf's observation that weighted voting does not grant people equal impact on the electoral college. The results also show that the amount of power the highest person has in the electoral college has increased in the past few decades since Banzhaf published "One Man, 3.312 Votes" from 3.312 (New York in the 1960s) to 3.4928 votes (California in the 2020s). While the electoral votes in New York in the 1960s was 54 and California now has 55, there is nearly two-tenths of an increase in the weight of one resident's votes. We do not have a reason for this large increase (though a theory could be the rapid change in population), leaving another possibility for research.

Contrary to popular belief, the electoral college does not award the lesser populated states with more votes, instead, those in more populous states have almost 3.5 times more weight. Mathematically, the reasoning behind the electoral college does not match up with the political reasoning of its intent [13]. Instead of distributing power more equally to those with smaller populations, the electoral college does the opposite. Though there is renewed philosophical and political interest in the electoral college, it is important to examine if the electoral college is fulfilling its purpose. Our results reveal that it is ineffective almost to the point of undermining the intent of the electoral college.

Examining the Banzhaf Ratio (which indicates the strength of a state to the weakest state regardless of population), we find that California has 20.3 times more weight than DC, despite the electoral ratio being 55:3 or approximately 18.3. When ignoring population, California should be thought to have influence equal to that of its ratio of electors to other states. Instead, the electoral college gives California more of an advantage. This advantage is also seen when looking at the values for the Normalized Banzhaf Power Index. In our simulations, North Dakota results as the weakest state while California has the largest value. The ratio of these values again exceeds the ratio of electoral votes, equal to around 20.3.

When looking at the Relative Strength across voter populations, California gets another boost in weight though DC falls in impact. There is no explanation for this as the electoral college seems to be awarding more populous states, but here, where DC has a larger voter registration and turnout by percentage of population [21], California benefits. This seems counter-intuitive because having a higher voting registration or voter turnout does not seem to better one's chances at impacting the presidential election. This implies that abstaining from voting increases the weight of other voters in the same state. Instead of encouraging people to vote, there is the possibility that this data may be used to manipulate eligible voters if the electoral assignment process changes to be based on voter registration or turnout. Though this is unlikely to happen, it is necessary to consider the ways in which the Banzhaf Relative Strength based on standard population may stir a change in the ways in which we view elections.

This study finds that all states are not equal nor are all voters equal. The notion that presidential campaigns should focus on states that maximize their votes has mathematical validity. While many presidential campaigns target states due to ideological reasons, demographics or the cycle of the election, there is the chance that campaigns incorporate the mathematical weight of each state as an additional factor to campaign planning. Swing states are often targeted based on a combination of these factors, and once people begin to understand that all states are not equal, it is possible that these swing states may change.

There are many vulnerabilities in the electoral college and it is necessary to contemplate how our study's data can both be used to improve upon the electoral college or prey upon it. The purpose of this study was to better understand if the electoral college is mathematically sound, and that has been answered. Our research shows that the electoral college fails to equalize voting across the country, and as quoted in *Baker V. Carr* by Justice Brennan: "A citizen's right to a vote free of arbitrary impairment by state action has been judicially recognized as a right secured by the Constitution, when such impairment resulted from dilution by a false tally... or by a refusal to count votes from arbitrarily selected precincts... or by a stuffing of the ballot box" [3]. While *Baker v. Carr* establishes the "one man, one vote" policy, it is evident that the electoral college violates this policy. The electoral college does not seem to offer any explanations, behaving the opposite of what is expected and even what the creators intended [13]. It does not grant smaller states more power- rather, the mathematics seem to suggest a debasement residents from smaller states. If the electoral college is supposed to create equality amongst votes, it falls short by 2.492 votes. Though the electoral college is frequently touted as a pillar of federalism, allowing states to have some control over national affairs, less populous states might wonder if they are getting the short end of the stick.

6 Conclusion

Though much debate about the electoral college revolves around principles, there is the simple mathematical result that the electoral college is not effective in distributing power. When contemplating a complex topic like the electoral college, mathematics is a tool to evaluate if the intent and the results match, showing that there many disconnects exist. This study reveals that though much is left unknown about the electoral college, careful contemplation of modifications to the electoral college should not be discounted on belief that the electoral college supports smaller states. The examination of alternative voting methods is necessary when it is clear that the voting method is found to violate the Constitutional right to vote. This is no different than the command to redistrict in *Baker v. Carr*, that it is unconstitutional to allow a flawed voting system to exist, perhaps an even more urgent as it is the election of our nation's highest office.

This study is limited in that it is not an exact calculation of the power of each state due to time and technology constraints. The research does offer current calculations of 150 million elections with the most recent Census. More importantly, this is the first study to consider how voting populations affect the result of the electoral college, introducing that the electoral college rewards states with a lower voter registration percentage. The paper is a first step, and a number of future studies can be carried out to provide a clearer picture on the intricacies of the electoral college. These include: the tendencies to vote certain ways based on location, race, and other demographics as an impact on a citizen's vote, historical analysis of changes in states' Banzhaf values, voting blocs as reducing or increasing the power of states to swing the election their way, analyzing a district-elector system as seen in Maine and Nebraska, and a historical analysis of campaign spending to see if candidates made a worthwhile investment and return.

The recommendations of this study are to consider alternative voting methods. People need to consider if the electoral college can be reformed, such as by following Nebraska and Maine's district-elector system and making electors bound to follow the plurality vote of the state. If people believe that the electoral college is beyond repair, voting systems such as preferential, majority, plurality (popular), two-round system, and many others can be considered. Though some may argue that reforming the electoral college violates the US Constitution, it can also be argued that the electoral college violates the right to vote by debasing the votes of certain residents. Reforming the election process may require changes to the US Constitution or states' Constitutions, yet reformation benefits the fundamental democracy in which the Constitutions owe their existence to. Democracy is contingent on the voice of the people, heard loudest on Election Day, but the electoral college fails to be a microphone to the very people it exists to protect.

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8 Supplementary Materials

Table 1. Additional Banzhaf Metrics

	Electoral Votes	Swings	Absolute	Normalized
Alabama	9	687502.79	0.068757	0.016449
Alaska	3	228511.21	0.022854	0.005466
Arizona	11	841179.29	0.084110	0.020118
Arkansas	6	457592.29	0.045757	0.010944
California	54	4642562.64	0.464276	0.111049
Colorado	10	763863.00	0.076386	0.018271
Connecticut	7	534045.00	0.053412	0.012773
Delaware	3	228557.86	0.022857	0.005468
DC	3	228434.21	0.022845	0.005465
Florida	30	2347316.93	0.234749	0.056146
Georgia	16	1227640.07	0.122762	0.029363
Hawaii	4	304733.86	0.030472	0.007290
Idaho	4	285348.21	0.028668	0.006786
Illinois	19	1462804.14	0.146282	0.034984
Indiana	11	841074.57	0.084104	0.020118
Iowa	6	457583.14	0.045758	0.010945
Kansas	6	457705.29	0.045771	0.010946
Kentucky	8	610789.71	0.061078	0.014612
Louisiana	8	610790.43	0.061081	0.014614
Maine	4	304903.14	0.030486	0.007292
Maryland	10	764298.93	0.076431	0.018284
Massachusetts	11	841376.07	0.084131	0.020124
Michigan	15	1149945.79	0.114997	0.027505
Minnesota	10	764521.93	0.076446	0.018285
Mississippi	6	457572.50	0.045756	0.010946
Missouri	10	764165.00	0.076415	0.018278
Montana	4	304733.86	0.030473	0.007289
Nebraska	5	386819.50	0.038640	0.009262
Nevada	6	452421.50	0.045280	0.010814
New Hampshire	4	315947.14	0.031517	0.007576
New Jersey	14	1072770.14	0.107277	0.025655
New Mexico	5	356273.93	0.035793	0.008473
New York	28	2182584.43	0.218264	0.052204
North Carolina	16	1227655.36	0.122774	0.029363
North Dakota	3	374947.79	0.036517	0.005464
Ohio	17	1305809.71	0.130584	0.031231
Oklahoma	7	534073.86	0.053408	0.012777
Oregon	8	610858.71	0.061086	0.014612
Pennsylvania	19	1462761.79	0.146283	0.034986
Rhode Island	4	304872.71	0.030486	0.007293
South Carolina	9	687454.07	0.068746	0.016442
South Dakota	3	228517.57	0.022854	0.005468
Tennessee	11	841364.29	0.084130	0.020123
Texas	40	3199281.07	0.319920	0.076519
Utah	6	457535.50	0.045755	0.010943
Vermont	3	228666.86	0.022870	0.005472
Virginia	13	995111.86	0.099513	0.023803
Washington	12	917960.86	0.091804	0.021957
West Virginia	4	304746.57	0.030474	0.007289
Wisconsin	10	714948.14	0.071826	0.016992
Wyoming	3	228643.50	0.022865	0.005470

Table 2. Expected Error

	s Normalized	s^2 Normalized	s Absolute	s^2 Absolute	s Swings	s^2 Swings
Alabama	0.0002283315	0.0000000521	0.0000835463	0.0000000070	835.463	697999
Alaska	0.0000735329	0.0000000054	0.0000321952	0.0000000010	321.952	103653
Arizona	0.0002777759	0.0000000772	0.0000921283	0.0000000085	921.283	848762
Arkansas	0.0001456002	0.0000000212	0.0000618772	0.0000000038	618.772	382878
California	0.0014960120	0.0000022381	0.0001562223	0.0000000244	1562.223	2440541
Colorado	0.0002448673	0.0000000600	0.0000931957	0.0000000087	931.957	868544
Connecticut	0.0001618475	0.0000000262	0.0000739096	0.0000000055	739.096	546263
Delaware	0.0000733992	0.0000000054	0.0000369171	0.0000000014	369.171	136287
DC	0.0000765350	0.0000000059	0.0000296384	0.0000000009	296.384	87843
Florida	0.0007577673	0.0000005742	0.0001359811	0.0000000185	1359.811	1849086
Georgia	0.0003914706	0.0000001532	0.0001137365	0.0000000129	1137.365	1293598
Hawaii	0.0001015687	0.0000000103	0.0000469523	0.0000000022	469.523	220452
Idaho	0.0016860874	0.0000028429	0.0070736436	0.0000500364	70736.436	5003643417
Illinois	0.0004670712	0.0000002182	0.0000997963	0.0000000100	997.963	995929
Indiana	0.0002780037	0.0000000773	0.0000791232	0.0000000063	791.232	626048
Iowa	0.0001482729	0.0000000220	0.0000489729	0.0000000024	489.729	239834
Kansas	0.0001456030	0.0000000212	0.0000564165	0.0000000032	564.165	318282
Kentucky	0.0002020062	0.0000000408	0.0000769389	0.0000000059	769.389	591959
Louisiana	0.0002053760	0.0000000422	0.0000760139	0.0000000058	760.139	577811
Maine	0.0000997650	0.0000000100	0.0000603870	0.0000000036	603.870	364659
Maryland	0.0002527417	0.0000000639	0.0000922657	0.0000000085	922.657	851296
Massachusetts	0.0002748608	0.0000000755	0.0000910233	0.0000000083	910.233	828524
Michigan	0.0003697846	0.0000001367	0.0001000337	0.0000000100	1000.337	1000674
Minnesota	0.0002408438	0.0000000580	0.0000854507	0.0000000073	854.507	730182
Mississippi	0.0001502900	0.0000000226	0.0000860247	0.0000000074	860.247	740025
Missouri	0.0002464836	0.0000000608	0.0000935689	0.0000000088	935.689	875514
Montana	0.0000974863	0.0000000095	0.0000423141	0.0000000018	423.141	179048
Nebraska	0.0004820375	0.0000002324	0.0019836727	0.0000039350	19836.727	393495722
Nevada	0.0005027677	0.0000002528	0.0019854290	0.0000039419	19854.290	394192839
New Hampshire	0.0009406094	0.0000008847	0.0039535637	0.0000156307	39535.637	1563066594
New Jersey	0.0003440015	0.0000001183	0.0000777801	0.0000000060	777.801	604975
New Mexico	0.0021519318	0.0000046308	0.0090282451	0.0000815092	90282.451	8150920880
New York	0.0007034738	0.0000004949	0.0000924515	0.0000000085	924.515	854729
North Carolina	0.0003928830	0.0000001544	0.0000738910	0.0000000055	738.910	545988
North Dakota	0.0119882697	0.0001437186	0.0529598078	0.0028047412	529598.078	280474123820
Ohio	0.0004217996	0.0000001779	0.0001044734	0.0000000109	1044.734	1091470
Oklahoma	0.0001762112	0.0000000311	0.0000479906	0.0000000023	479.906	230310
Oregon	0.0002006164	0.0000000402	0.0000686917	0.0000000047	686.917	471855
Pennsylvania	0.0004730397	0.0000002238	0.0001059410	0.0000000112	1059.410	1122350
Rhode Island	0.0001017241	0.0000000103	0.0000498342	0.0000000025	498.342	248345
South Carolina	0.0002208774	0.0000000488	0.0000550978	0.0000000030	550.978	303577
South Dakota	0.0000742111	0.0000000055	0.0000398686	0.0000000016	398.686	158951
Tennessee	0.0002771717	0.0000000768	0.0001056636	0.0000000112	1056.636	1116480
Texas	0.0010322300	0.0000010655	0.0001227371	0.0000000151	1227.371	1506439
Utah	0.0001458208	0.0000000213	0.0000539264	0.0000000029	539.264	290806
Vermont	0.0000773632	0.0000000060	0.0000477045	0.0000000023	477.045	227572
Virginia	0.0003196340	0.0000001022	0.0000597517	0.0000000036	597.517	357027
Washington	0.0002922139	0.0000000854	0.0000588716	0.0000000035	588.716	346587
West Virginia	0.0000999579	0.0000000100	0.0000436226	0.0000000019	436.226	190293
Wisconsin	0.0042247770	0.0000178487	0.0177581154	0.0003153507	177581.155	31535066418
Wyoming	0.0000767131	0.0000000059	0.0000384618	0.0000000015	384.618	147931
Average	0.0006787004	0.0000034741	0.0019222327	0.0000642241		
Monte Carlo Error		0.0000003296%		0.0000060928%		

Table 3. Population Data

	Standard (Pop. A)	Voter Registration (Pop. B)*	Voter Turnout (Pop. C)*
Alabama	5,024,279	2,527,000	2,247,000
Alaska	733,391	383,000	330,000
Arizona	7,151,502	3,878,000	3,649,000
Arkansas	3,011,524	1,361,000	1,186,000
California	39,538,223	18,001,000	16,893,000
Colorado	5,773,714	2,993,000	2,837,000
Connecticut	3,605,944	1,850,000	1,681,000
Delaware	989,948	542,000	489,000
DC	689,545	464,000	448,000
Florida	21,538,187	10,495,000	9,720,000
Georgia	10,711,908	5,233,000	4,888,000
Hawaii	1,455,271	673,000	630,000
Idaho	1,839,106	900,000	843,000
Illinois	12,812,508	6,590,000	6,058,000
Indiana	6,785,528	3,412,000	3,002,000
Iowa	3,190,369	1,742,000	1,618,000
Kansas	2,937,880	1,398,000	1,297,000
Kentucky	4,505,836	2,450,000	2,210,000
Louisiana	4,657,757	2,286,000	2,041,000
Maine	1,362,359	832,000	766,000
Maryland	6,177,224	3,383,000	3,166,000
Massachusetts	7,029,917	3,546,000	3,249,000
Michigan	10,077,331	5,513,000	4,994,000
Minnesota	5,706,494	3,436,000	3,225,000
Mississippi	2,961,279	1,749,000	1,531,000
Missouri	6,154,913	3,388,000	2,990,000
Montana	1,084,225	641,000	607,000
Nebraska	1,961,504	971,000	892,000
Nevada	3,104,614	1,455,000	1,351,000
New Hampshire	1,377,529	843,000	797,000
New Jersey	9,288,994	5,008,000	4,638,000
New Mexico	2,117,522	1,028,000	938,000
New York	20,201,249	9,370,000	8,609,000
North Carolina	10,439,388	5,161,000	4,780,000
North Dakota	779,094	429,000	373,000
Ohio	11,799,448	6,733,000	6,128,000
Oklahoma	3,959,353	1,884,000	1,631,000
Oregon	4,237,256	2,590,000	2,402,000
Pennsylvania	13,002,700	7,337,000	6,756,000
Rhode Island	1,097,379	575,000	515,000
South Carolina	5,118,425	2,713,000	2,459,000
South Dakota	886,667	437,000	380,000
Tennessee	6,910,840	3,742,000	3,346,000
Texas	29,145,505	13,343,000	11,874,000
Utah	3,271,616	1,468,000	1,386,000
Vermont	643,077	365,000	342,000
Virginia	8,631,393	4,541,000	4,275,000
Washington	7,705,281	4,029,000	3,854,000
West Virginia	1,793,716	928,000	773,000
Wisconsin	5,893,718	3,391,000	3,253,000
Wyoming	576,851	296,000	280,000

From US Census Bureau [22],[23]

*Voter Registration and Voter Turnout given in thousands as given by US Census Bureau

Table 4. Rankings Across Banzhaf Metrics

	ν_i Rank	β_i Pop. A Rank	β_i Pop. B Rank	β_i Pop. C Rank	Average Rank
Alabama	23	23	19	18	20.75
Alaska	50	39	41	35	41.25
Arizona	16	20	20	21	19.25
Arkansas	32	41	29	28	32.5
California	1	1	1	1	1
Colorado	21	18	17	19	18.75
Connecticut	28	33	28	30	29.75
Delaware	48	49	50	50	49.25
DC	51	35	47	49	45.5
Florida	3	3	3	3	3
Georgia	9	10	9	9	9.25
Hawaii	44	46	40	41	42.75
Idaho	45	51	51	51	49.5
Illinois	6	6	5	6	5.75
Indiana	17	15	15	13	15
Iowa	31	44	43	45	40.75
Kansas	30	38	34	33	33.75
Kentucky	27	30	31	31	29.75
Louisiana	26	32	26	25	27.25
Maine	41	42	48	47	44.5
Maryland	19	22	23	24	22
Massachusetts	14	19	16	16	16.25
Michigan	10	11	11	11	10.75
Minnesota	18	17	25	26	21.5
Mississippi	33	40	44	43	40
Missouri	20	21	24	20	21.25
Montana	43	28	35	37	35.75
Nebraska	36	34	30	32	33
Nevada	35	43	39	39	39
New Hampshire	39	36	46	46	41.75
New Jersey	11	12	12	12	11.75
New Mexico	38	47	42	44	42.75
New York	4	4	4	4	4
North Carolina	8	9	7	8	8
North Dakota	37	5	6	5	13.25
Ohio	7	8	10	10	8.75
Oklahoma	29	37	33	29	32
Oregon	25	26	36	36	30.75
Pennsylvania	5	7	8	7	6.75
Rhode Island	40	29	27	27	30.75
South Carolina	24	24	22	22	23
South Dakota	49	48	45	42	46
Tennessee	15	16	18	17	16.5
Texas	2	2	2	2	2
Utah	34	45	38	40	39.25
Vermont	46	31	37	38	38
Virginia	12	13	13	14	13
Washington	13	14	14	15	14
West Virginia	42	50	49	48	47.25
Wisconsin	22	27	32	34	28.75
Wyoming	47	25	21	23	29

Pop. A Census Population
Pop. B Voter Registration
Pop. C Voter Turnout

Table 5. Banzhaf Metrics Based on 2020 Census

	Electoral Votes	Banzhaf Ratio ν_i	β_i Pop. A	β_i Pop. B	β_i Pop. C
Alabama	9	3.0097	1.4511	1.4313	1.4690
Alaska	3	1.0004	1.2624	1.2220	1.2741
Arizona	11	3.6817	1.4878	1.4134	1.4102
Arkansas	6	2.0029	1.2473	1.2979	1.3457
California	54	20.3225	3.4928	3.6212	3.6178
Colorado	10	3.3436	1.5038	1.4611	1.4525
Connecticut	7	2.3380	1.3306	1.2995	1.3194
Delaware	3	1.0005	1.0867	1.0274	1.0468
DC[1]	3	1.0000	1.3014	1.1099	1.0931
Florida	30	10.2756	2.3928	2.3979	2.4115
Georgia	16	5.3736	1.7743	1.7759	1.7783
Hawaii	4	1.3338	1.1949	1.2292	1.2296
Idaho[2,3,4]	4	1.2549	1.0000	1.0000	1.0000
Illinois	19	6.4031	1.9332	1.8857	1.9035
Indiana	11	3.6814	1.5273	1.5067	1.5546
Iowa	6	2.0029	1.2119	1.1473	1.1521
Kansas	6	2.0035	1.2632	1.2810	1.2872
Kentucky	8	2.6736	1.3612	1.2913	1.3159
Louisiana	8	2.6737	1.3388	1.3369	1.3693
Maine	4	1.3344	1.2355	1.1060	1.1156
Maryland	10	3.3456	1.4547	1.3751	1.3757
Massachusetts	11	3.6826	1.5010	1.4785	1.4948
Michigan	15	5.0337	1.7136	1.6208	1.6481
Minnesota	10	3.3462	1.5138	1.3647	1.3633
Mississippi	6	2.0029	1.2578	1.1449	1.1843
Missouri	10	3.3449	1.4571	1.3738	1.4153
Montana	4	1.3339	1.3844	1.2595	1.2527
Nebraska	5	1.6914	1.3051	1.2976	1.3103
Nevada	6	1.9820	1.2157	1.2422	1.2477
New Hampshire	4	1.3796	1.2703	1.1359	1.1306
New Jersey	14	4.6958	1.6651	1.5864	1.5954
New Mexico	5	1.5668	1.1636	1.1682	1.1836
New York	28	9.5540	2.2972	2.3596	2.3825
North Carolina	16	5.3741	1.7975	1.7884	1.7985
North Dakota	3	1.5985	1.9571	1.8450	1.9150
Ohio	17	5.7160	1.7983	1.6654	1.6895
Oklahoma	7	2.3378	1.2697	1.2876	1.3394
Oregon	8	2.6739	1.4038	1.2561	1.2623
Pennsylvania	19	6.4032	1.9190	1.7871	1.8025
Rhode Island	4	1.3345	1.3767	1.3304	1.3606
South Carolina	9	3.0092	1.4374	1.3812	1.4041
South Dakota	3	1.0004	1.1481	1.1441	1.1874
Tennessee	11	3.6826	1.5139	1.4392	1.4730
Texas	40	14.0037	2.8033	2.8983	2.9734
Utah	6	2.0028	1.1966	1.2497	1.2447
Vermont	3	1.0011	1.3491	1.2527	1.2525
Virginia	13	4.3560	1.6023	1.5454	1.5415
Washington	12	4.0185	1.5645	1.5135	1.4977
West Virginia	4	1.3339	1.0764	1.0468	1.1101
Wisconsin	10	3.1440	1.3996	1.2907	1.2754
Wyoming	3	1.0009	1.4241	1.3908	1.3839

[1]Weakest Banzhaf Ratio
[2]Weakest Relative Strength Based on Census Population
[2]Weakest Relative Strength Based on Voter Registration
[3]Weakest Relative Strength Based on Voter Turnout

Pop A. Census Population
Pop B. Voter Registration
Pop C. Voter Turnout