WORKING PAPER

Where is the Post in First-Past-the-Post (and Beyond)?

A Logical Model of the Effective District-Wide Threshold

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ABSTRACT In this short note, I set out to answer a deceptively simple question: where exactly is the post in first-past-the-post and other electoral systems? To this end, I present a new model of the *effective district-wide threshold*: the vote share at which any given party has a 50/50 chance of winning its first seat in a district. To derive my model, I rely only on simple combinatorics. The resulting expression suggests both that, all else being equal, we should expect parties to be more likely than not to win at least one seat in a district and that the effective district-wide threshold depends only on the district's magnitude and the number of vote-winning parties. Using district-level data from 375 elections in 110 countries, I then estimate a non-linear Bayesian model that strongly corroborates my theoretical logic. As such, the model that I present here serves both to advance the broader Taageperan research agenda of interlocking electoral models and allows for new quasi-experimental research designs that exploit as-good-as-random variation around electoral thresholds.

1 INTRODUCTION

Democracies use elections to convert the will of the people into political power. Yet, different electoral systems often lead to different outcomes, especially when it comes to the representation of smaller and less popular parties (Duverger 1964). One notable obstacle that small parties face is the electoral threshold: the share of the vote necessary in order to gain representation. These thresholds can be explicit and written into the law. For instance, Spain requires that parties gain at least 3% in any district before they win a seat. In most cases, however, thresholds remain implicit and arise as a result of the distinct mechanics of the electoral system (Rae, Hanby, and Loosemore 1971). Still, whether they are explicit or they are not, the conclusion is the same: electoral thresholds serve to shape the party system.

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In this short note, I set out to answer a deceptively simple question: where exactly is the post in first-past-the-post and other systems? To this end, I derive a new model of the *effective district-wide threshold*—the point at which any given party has a 50/50 chance of winning its first seat in a district. When compared to past attempts (Taagepera 2002, 1998; Lijphart 1994), my model has many notable advantages. First, it uses fewer simplifying approximations or untested assumptions. As such, it should generalise well to a range of applications. Second, empirical evaluation of my model shows that real world data corroborate my logic almost exactly. Consequently, we can be confident that my model produces reliable predictions. What's more, this process also produces two substantive implications. First, it reveals that, all else being equal, parties have a better chance than not of winning a seat in any district, suggesting a root cause for Duverger's "mechanical effect" (Riker 1982; Duverger 1964). Second, as well as advancing the broader Taageperan research agenda (for recent examples, see Hanretty 2022; Shugart and Taagepera 2017), my model also provides a source of quasi-random variation. Students of electoral politics might, therefore, use its predictions to test how adding new seat-winning parties affects a host of different outcomes.

2 ELECTORAL THRESHOLDS

Though analogies are sometimes useful, they can also cause confusion. Consider the term "first-past-the-post" which, though now synonymous with elections in single-member districts, itself, in fact, derives from a wager common in horse-racing. Though widespread and understood, it obscures as much as it reveals. For one, the "post" in question is not even fixed in place: parties can win seats despite having very different vote shares. What's more, this sort of dynamic is not limited to elections in single-member districts. In fact, *all* electoral systems enforce similar restrictions. One clear question, then, is where this threshold tends to occur.

At any district-level election, parties face two electoral cut-offs (Rae, Hanby, and Loosemore 1971). The first—the *threshold of representation*, t_R—reflects the *smallest* vote share that a party can receive while still having the chance to win a seat in a district. For example, in a single-member district with four vote-winning parties, the threshold of representation occurs where all parties receive near-equal support, save for one party that receives an additional vote (i.e. $t_R = 25\% + 1$). The second—the *threshold of exclusion*, t_X—reflects the *largest* vote share that a party can receive while still having the chance *not* to win a seat in a district. Consider another election under a single-member plurality system, though this time with only two vote-winning parties. In this case, the threshold of exclusion occurs where both parties win an almost share of the vote, save for one party having one fewer vote (i.e. $t_X = 50\% - 1$).

For any electoral district with a district magnitude of m and n_{V0} vote-winning parties, Taagepera and Shugart (1989) show that, in all simple electoral systems,[1] the threshold of representation, t_R, must be greater than or equal to the lower threshold of the Hare formula:

[1]"Simple" electoral systems allocate all seats within districts, use categorical ballots, and take place in a single round (see Shugart and Taagepera 2017). So, for example, first-past-the-post and any districted system of proportional representation are simple, but mixed-member systems or ranked-choice voting are not.

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$$t_R = \frac{1}{mn_{v0}} \tag{1}$$

Likewise, they also show that the threshold of exclusion, t_X, again in any district with a magnitude of m in a simple electoral system, must be less than or equal to the Droop quota:[2]

$$t_X = \frac{1}{m+1} \tag{2}$$

We can think of these two cut-offs as setting hard bounds on the *effective district-wide threshold*, t: the point at which any party has a 50/50 chance of winning their first seat in a district and which most closely approximates the "post" in first-past-the-post and other electoral systems. Since the thresholds of representation and exclusion represent lower- and upper-bounds on the effective threshold, respectively, we know that $t_R \leq t \leq t_X$. However, we do not know exactly where t might lie between them. At present, our best guess comes from Taagepera (2002), who models the effective district-wide threshold as follows:

$$t = \frac{0.75}{m+1} \tag{3}$$

Taagepera derives his model based on earlier work by Lijphart (1994), who estimates the effective district-wide threshold by taking the arithmetic mean of the thresholds of representation and exclusion subject to two key assumptions: that the number of vote-winning parties, n_{v0}, equals the district magnitude, m,[3] and that the effective threshold lies exactly halfway between each boundary threshold. This produces the following equation:

$$t = \frac{0.5}{m+1} + \frac{0.5}{2m} \tag{4}$$

Which Lijphart notes that we can rewrite as:

$$t = \frac{0.25\left(3 + \frac{1}{m}\right)}{m+1} \tag{5}$$

And which, if we omit $\frac{1}{m}$ in the numerator on the basis that it matters only for small values of m (see Lijphart 1994, 183–84), reduces to Taagepera's (2002) model as shown in Equation 3.

While Taagepera's (2002) work may represent our best guess of the effective district-wide threshold at present, this model is not without criticism (see, for example, Bischoff 2009). One clear problem that it faces stems from the simplifying assumption that the number of vote-winning parties, n_{v0}, equals the district magnitude, m. To see why, consider a simple contest between two parties in a single-member district (i.e. $n_{v0} = 2$ and $m = 1$). Here, both the threshold of representation and the threshold of exclusion (shown in Equations 1 and 2, respectively) are equal to 0.5. The effective district-wide threshold *must*, therefore, in this

[2]Since single-member district plurality or first-past-the-post systems represent a limiting case of both quota- and largest remainders-based PR systems (Shugart and Taagepera 2017), the same thresholds apply to these systems.

[3]Though I prefer to preserve both the number of vote-winning parties and the district magnitude rather than assume that they are equal, this is not an unreasonable assumption to make. In general, we might expect around $m + 1$ parties to contest a district with a magnitude of m (Cox 1997).

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case, also equal 0.5. Taagepera's equation, however, suggests that it is 0.375. Indeed, in any case where either n_{V0} or m are small, its predictions will likely be wrong. Furthermore, the assumption that the effective threshold, t, lies *exactly* half-way between the thresholds of representation, t_R, and exclusion, t_X, remains to be empirically-verified and, as we will see in the next section, there is good reason to think that it is false.

3 ESTIMATING THE EFFECTIVE DISTRICT-WIDE THRESHOLD

Imagine some electoral district, though you know nothing else about the geographical area that it covers, the people who live there, or their particular customs. All you know is that it has a magnitude of m and n_{V0} vote-winning parties. Given this assumption of ignorance, how likely should you think that it is for some party to win *at least one* seat in the district? Absent any other information, you should assume a uniform distribution over party support such that each party has a $\frac{1}{n_{V0}}$ chance of winning each seat, m_i. We can then use a useful feature of probability theory—that the chance of *at least one success* is the same as the chance that *not all trials are failures*—to compute the resulting probability. If we know that there is a $\frac{1}{n_{V0}}$ chance that a party *will* win a seat, that implies that there is a $1 - \frac{1}{n_{V0}}$ chance that it will not. As such, the chance that it will fail to win *any* of the district's m seats must be $\left(1 - \frac{1}{n_{V0}}\right)^m$, giving the chance that this *will not* happen, and that any party will win *at least one single* seat, as:

$$\Pr(m_i \geq 1) = 1 - \left(1 - \frac{1}{n_{V0}}\right)^m \tag{6}$$

One interesting property of this equation is that, in the limit where m and n_{V0} tend to ∞, the probability that any given party will win at least one seat in a district converges on $1 - \frac{1}{e} \approx 0.63$, where $e \approx 2.71$ is Euler's constant. So, to put it more plainly, with a sufficiently large district magnitude and a sufficiently large number of vote-winning parties, we should expect any given party to have around a 63% chance of winning at least one seat in a district, all else being equal. Further, as we can see from Figure 1, which plots the respective probabilities from Equation 6 as n_{V0} and m jointly increase in size, this probability converges on $1 - \frac{1}{e}$ very quickly, even for low values of n_{V0} and m. This suggests a potential root cause of Duverger's (1964) mechanical effect: the tendency of proportional electoral systems to feature more seat-winning parties. If parties have a greater than even chance of gaining representation, then, as party numbers proliferate, party-system fragmentation will grow.

Clearly, this combinatorial logic is at odds with Lijphart's (1994) and Taagepera's (2002) assumption that the thresholds of representation and exclusion deserve equal weight. Instead, it suggests that we should assign a weight of $1 - \frac{1}{e} \approx 0.63$ to the threshold of exclusion, t_X, since it reflects the point at which a party is *guaranteed* to win its first seat, and that, for the opposite reason, we should assign a weight of $1 - \left(1 - \frac{1}{e}\right) \approx 0.37$ to the threshold of representation, t_R. If we update Lijphart's (1994) model, shown in Equation 4, to account for these unequal weights, we arrive at the following logical model of the effective district-wide threshold:

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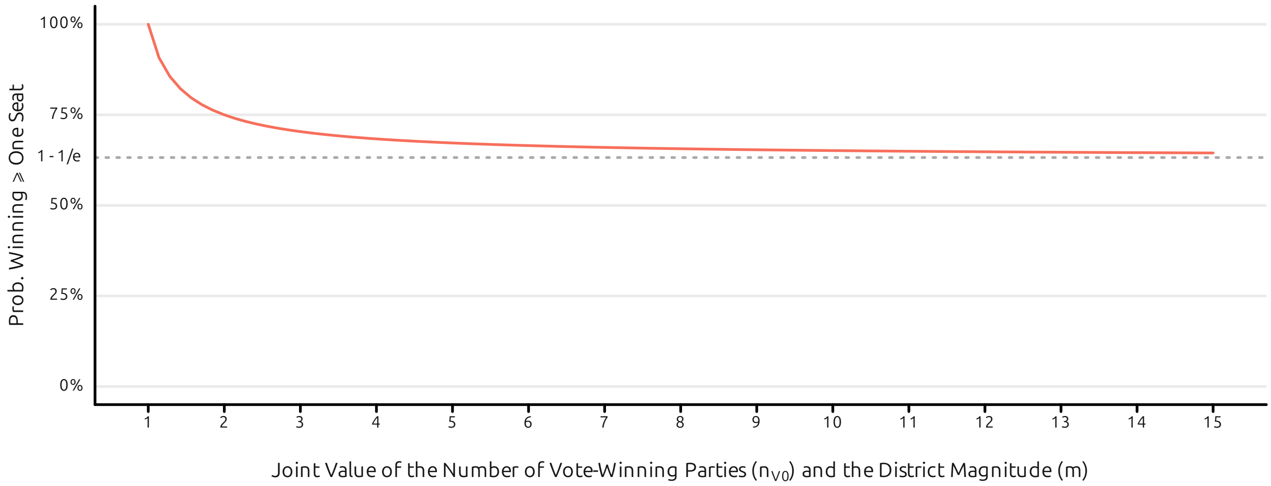

Figure 1: As the district magnitude and the number of vote-winning parties jointly increase in size, the probability that any given party will win at least one seat in a district tends to 1 - 1/e ≈ 0.63. This then implies that, all else being equal, parties have a better than even chance of winning a seat in districted systems of proportional representation.

$$t = \frac{1 - \left(1 - \frac{1}{e}\right)}{mn_{V0}} + \frac{1 - \frac{1}{e}}{m + 1} \tag{7}$$

Given the assumptions that I lay out above, this model represents our best estimate of the true effective district-wide threshold *in the absence of all other data*. As such, it remains possible that reality might disagree with the argument that I have set out. To test this possibility, I draw on district-level data from the Constituency-Level Elections Archive (Kollman et al. 2024). After removing errant cases—for example, which report districts as having a magnitude of zero—I am left with 143,552 observations, each representing the vote and seat share of some party in some district at one of 375 elections in one of 110 different countries. I then use the `brms` package (Bürkner 2017) in `R` to specify a non-linear Bayesian model that tests the logic that I lay out above. In particular, I fit the following model to my data:

$$w_i \sim \text{Bernoulli}(\pi_i) \tag{8}$$

$$\text{logit}(\pi_i) = e^{\beta} \left[\text{logit}(v_i) - \text{logit}(t_i)\right] \tag{9}$$

$$t_i = \frac{1 - \theta}{mn_{V0}} + \frac{\theta}{m + 1} \tag{10}$$

Though this may look somewhat complex, the equations above describe a simple two-parameter logistic model that we might tend to associate with an item-response theoretic approach (see de Ayala 2009), save for three small adjustments. First, I exponentiate the model's discrimination parameter, β, which determines how well the model distinguishes cases where party i did ($w_i = 1$) or did not ($w_i = 0$) win at least one seat in the district, for the sake of identification. Second, I put party i's vote share, v_i, and my estimate of the effective district-wide threshold in party i's district, t_i, on the logit scale to ensure that the model cannot

Table 1: My argument suggests that the mixing parameter, θ, should equal around 1 - 1/e ≈ 0.63. Remarkably, this is exactly the case, suggesting that the logic that I set out is correct. Data here come from district-level election results from the Constituency-Level Elections Archive.

	Estimate	Error	2.5%	97.5%
Discrimination, β	1.54	0.0087	1.53	1.56
Weight, θ	0.63	0.0023	0.63	0.64
Bayesian R^2	0.78	7e-04	0.77	0.78
N. Observations	143,552			

produce impossible negative estimates. Third, and finally, I model the effective district-wide threshold, t_i, using the functional form shown in Equation 7, though substitute $1 - \frac{1}{e}$ for the mixing parameter θ, which estimates the weight to assign to the threshold of exclusion, t_X. Assuming that my logic is correct, θ should equal around $1 - \frac{1}{e} \approx 0.63$.[4]

Remarkably, Table 1 confirms that this is *exactly* the case: the weighting parameter, θ, equals $1 - \frac{1}{e} \approx 0.63$ to two decimal places, strongly suggesting that the logic that I set out is correct. The model also fits well to the data and has a Bayesian R^2 of 0.78, suggesting that the model explains a good deal of the variation in the probability that a given party will win at least one seat in a district. Further, as the predictions in Figure 2 make clear, the model behaves much as we would expect. As we can see, in this case where the number of vote-winning parties is fixed such that $n_{v0} = 2$, as the district magnitude increases from $m = 2$ through $m = 3$ to $m = 5$, the effective district-wide threshold decreases from 50% to 22% to 14%, thereby becoming much more permissive. Likewise, Figure 2 also makes clear that the model's discrimination between cases with a high and low probability of winning representation is relatively steep, especially as the district magnitude grows. So, while the effective district-wide threshold may not be a *sharp* discontinuity, it still might not be overly fuzzy.

4 CONCLUSION

In this short note, I have set out to answer where the "post" lies in first-past-the-post and other electoral systems. To do so, I have derived a logical model of the relationship between a district's magnitude, m; the number of vote-winning parties that it contains, n_{v0}; and its effective threshold: the point at which any given party has a 50/50 chance of winning its first seat in the district. Though real world data corroborate my logic empirically, its derivation does not rely on data alone. Rather, it is based only on simple combinatorics.

My findings have implications for the study of electoral politics. First, much like recent work by Hanretty (2022) and Shugart and Taagepera (2017), they serve to advance the Taageperan research agenda by providing yet another parsimonious interlocking model of a specific parameter inherent to electoral systems. Logically, future research should seek to establish the equivalent quantity at the national level. Second, my findings also have the

[4]Note that I also include weakly-informative priors on the model's two free parameters, β and θ. For more information, including all relevant code, see the replication package that accompanies this note.

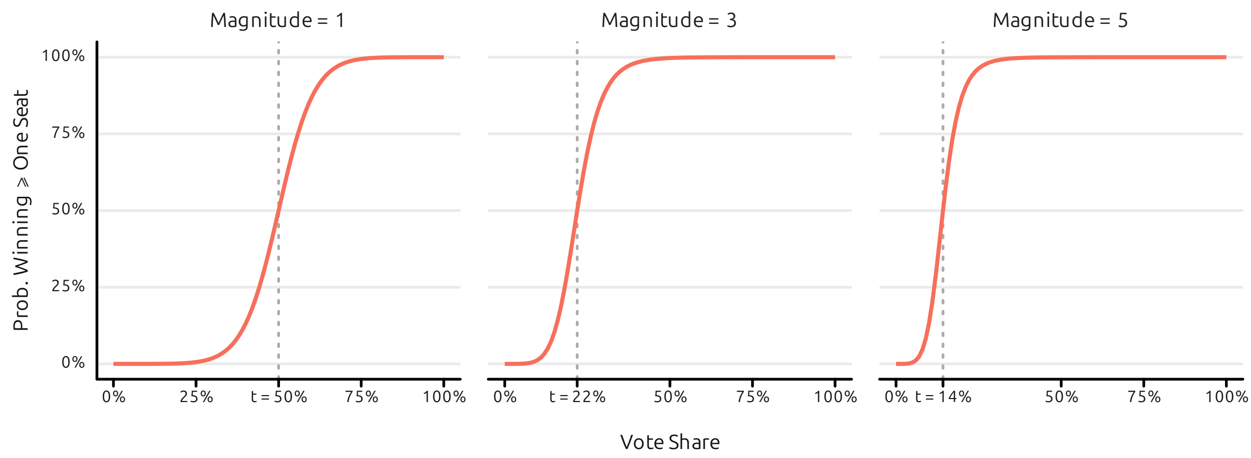

Figure 2: If we keep the number of vote-winning parties constant, in this case equal to 2, and increase the district magnitude, the effective district-wide threshold decreases in turn, thereby causing the electoral system to become much more permissive.

potential to unlock new forms of quasi-random variation. Recent research by Matsunaga and Winzen (2024), Valentim and Dinas (2024), Abou-Chadi and Krause (2020), and others has used Taagepera's (2002) model to exploit the quasi-random assignment to seat-winning parties that occurs around electoral thresholds. Since my model makes more accurate predictions and has stronger empirical backing, it should also improve our precision. So, with any luck, it should reveal new avenues for causal inference in electoral studies.

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