WORKING PAPER

Where’s the Post in First-Past-the-Post (and Beyond)?

A Logical Model of the Effective District-Wide Threshold

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ABSTRACT In this short note, I set out to answer a deceptively simple question: where does the “post” lie in first-past-the-post and other electoral systems? To this end, I present a new logical model of the *effective district-wide threshold*: the vote share at which any given party has a 50/50 chance of winning its first seat in a district. To derive my model, I rely only on simple combinatorics. The resulting expression suggests both that, all else being equal, we should expect parties to be more likely than not to win at least one seat in a district and that the effective district-wide threshold depends only on the district’s magnitude and the number of vote-winning parties. Using district-level data from 375 elections in 110 countries, I then estimate a non-linear Bayesian model that strongly corroborates my theoretical logic. As such, the model that I present here advances the broader Taageperan research agenda of interlocking electoral models and may also allow for new quasi-experimental research designs that exploit as-good-as-random variation around electoral thresholds.

1 INTRODUCTION

Democracies use elections to convert *the will of the people* into *political power*. Yet, different electoral systems can lead to different outcomes, especially when it comes to electing smaller and less popular parties (Duverger 1964). One notable obstacle that many small parties face is the *electoral threshold*: the share of the vote that parties must receive in order to win a seat in the legislature. In some cases, these thresholds are explicit and written into the law. Spain, for instance, mandates that parties gain at least 3% in a district before they may win a seat. In other cases, however, thresholds remain implicit instead and arise due to the mechanics of the electoral system (Rae, Hanby, and Loosemore 1971). Whatever the case, the conclusion remains much the same: electoral thresholds shape party systems.

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In this note, I present a new quantitative logical model (Taagepera 2008) of the *effective district-wide threshold*—the point at which any given party has a 50/50 chance of winning its first seat in a district. In effect, this provides an answer to a deceptively simple question: where does the post lie in first-past-the-post and other electoral systems? When compared to past attempts (Taagepera 2002, 1998; Lijphart 1994), my model has two clear advantages: it uses fewer simplifying approximations or untested assumptions and data from the real world corroborate its theoretically-derived parameters almost exactly. Consequently, we can be confident that my model will produce reliable predictions and that it will generalise well to a range of electoral systems. As well as advancing the broader Taageperan research agenda of parsimonious and interlocking electoral models (for recent examples, see Hanretty 2022; Shugart and Taagepera 2017), the model that I present here also has broader implications for causal inference in electoral studies. In particular, it should allow future research to compute the propensity that any party will be assigned to a seat and to exploit the quasi-random and fuzzy assignment to seat-winning parties that occurs around effective electoral thresholds.

2 REPRESENTATION AND ELECTORAL THRESHOLDS

Though analogies are sometimes useful, they can also be somewhat confusing. Consider the term "first-past-the-post" which, though now synonymous with elections in single-member districts, comes from a type of wager in horse-racing. Though now widespread and well-understood, the term obscures as much as it reveals. For one, unlike in horse-racing, the "post" in elections is not fixed in place: parties can still win seats even if they have very different shares of the vote. What's more, elections in single-member districts are not the only type of election to have some sort of "post". Rather, *all* electoral systems enforce this type of restriction. One obvious question, then, is where these thresholds tend to occur.

In any district-level election, parties face two electoral cut-offs (Rae, Hanby, and Loosemore 1971). The first—the *threshold of representation*, t_R—reflects the *smallest* vote share that a party could win while still having a chance of winning a seat in a district. For example, in a single-member district with four vote-winning parties, the threshold of representation occurs where all parties receive near-equal support, save for one party that receives an additional vote (i.e. $t_R = 25\%+1$). The second—the *threshold of exclusion*, t_X—reflects the *largest* vote share that a party could win while still having the chance *not* to win a seat in a district. Consider again the previous case. Here, the threshold of exclusion would occur where one of the four parties won just under 50% of the vote, since another party could beat (i.e. $t_X = 50\% - 1$).

For any electoral district with a district magnitude of m and n_{V0} vote-winning parties, Taagepera and Shugart (1989, 276) show that, in all simple electoral systems,[1] the threshold of representation, t_R, must be greater than or equal to the lower threshold of the Hare formula:

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[1] "Simple" electoral systems allocate all seats within districts, use categorical ballots, and take place in a single round (see Shugart and Taagepera 2017). So, for example, first-past-the-post and any districted system of proportional representation are simple, but mixed-member systems or ranked-choice voting are not.

$$t_R = \frac{1}{m n_{v0}} \tag{1}$$

Likewise, they also show that the threshold of exclusion, t_X, again in any district with a magnitude of m in a simple electoral system, must be less than or equal to the Droop quota:[2]

$$t_X = \frac{1}{m+1} \tag{2}$$

As such, we can think of these two cut-offs as setting hard bounds on the *effective district-wide threshold*, t: the point at which any party has a 50/50 chance of winning their first seat in a district. Since the thresholds of representation and exclusion represent lower- and upper-bounds on the effective threshold, respectively, we know that $t_R \leq t \leq t_X$. However, we *do not* know exactly where t might lie between them. At present, our best guess comes from Taagepera (2002), who models the effective district-wide threshold as follows:

$$t = \frac{0.75}{m+1} \tag{3}$$

To derive his model, Taagepera draws on earlier work by Lijphart (1994), who estimates the effective district-wide threshold by taking the arithmetic mean of the thresholds of representation and exclusion subject to two key assumptions: that the number of vote-winning parties, n_{v0}, equals the district magnitude, m,[3] and that the effective district-wide threshold lies exactly halfway between each of the bounds. This produces the following equation:

$$t = \frac{0.5}{m+1} + \frac{0.5}{2m} \tag{4}$$

Which Lijphart notes that we can rewrite as:

$$t = \frac{0.25\left(3 + \frac{1}{m}\right)}{m+1} \tag{5}$$

And which, if we omit $\frac{1}{m}$ in the numerator on the basis that it matters only for small values of m (see Lijphart 1994, 183–84), reduces to Taagepera's (2002) model as shown in Equation 3.

While Taagepera's (2002) work may represent our best guess of the effective district-wide threshold at present, his model is not without criticism (see, for example, Bischoff 2009). One clear problem that it faces stems from the simplifying assumption that the number of vote-winning parties, n_{v0}, equals the district magnitude, m. To see why, consider a simple contest between two parties in a single-member district (i.e. $n_{v0} = 2$, $m = 1$). Here, both the threshold of representation and the threshold of exclusion (shown in Equations 1 and 2, respectively) equal exactly 50%. The effective district-wide threshold *must*, therefore, be 50% in this case. Taagepera's equation, however, suggests that it is 37.5%. Indeed, in any case where either n_{v0}

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[2] Since first-past-the-post represents a limiting case of both quota- and largest remainders-based PR systems (Shugart and Taagepera 2017), the same thresholds also apply to this system.

[3] Though I prefer to preserve both the number of vote-winning parties and the district magnitude rather than assume they are equal, this is not an unreasonable assumption. As Cox (1997) argues, we might expect around $m + 1$ parties to contest a district with a magnitude of m.

or m are small, the model's predictions will likely be wrong. Furthermore, the assumption that the effective threshold, t, lies *exactly* half-way between the thresholds of representation, t_R, and exclusion, t_X, remains to be tested empirically and, as we will see in the next section, there is good reason to believe that it is false.

3 MODELLING THE EFFECTIVE DISTRICT-WIDE THRESHOLD

Since the thresholds of representation, t_R, and exclusion, t_X, are non-linear functions of the district magnitude, m, and the number of vote-winning parties, n_{V0}, any model of the effective district-wide threshold, t, must be non-linear too. As such, I follow a two-step approach informed by Taagepera (2008) and Shugart and Taagepera (2017), who advocate in favour of theoretically-informed, parsimonious, and interlocking non-linear models of electoral systems. In the first step, I use simple combinatorics to derive a theoretically-informed logical model that predicts what we should expect the effective district-wide threshold to be *in the absence of all other data*. In the second step, I then specify a similar non-linear Bayesian model using the `brms` package (Bürkner 2017) in `R` that puts my logic to the test.

3.1 THEORETICAL MODEL

To derive my logical model of the effective district-wide threshold, it is necessary to first imagine some electoral district. However, assume that you know *nothing* about the area it covers, the people who live there, or their customs. Assume that you know *only* that it has a district magnitude of m and features n_{V0} vote-winning parties. Given this assumption of ignorance, how likely is it that some party will win *at least one* seat in the district? Absent any other information, you should assume that all n_{V0} vote-winning parties have equal support such that each party has a $\frac{1}{n_{V0}}$ chance of winning each seat, m_i. You should then use a useful feature of probability theory—that the chance of *at least one success* is the same as the chance that *not all trials are failures*—to compute the resulting probability. If there is a $\frac{1}{n_{V0}}$ chance that any party *will* win a seat, there is a $1 - \frac{1}{n_{V0}}$ chance that it *will not*. Thus, the chance that it will fail to win *any* of the district's m seats must be $\left(1 - \frac{1}{n_{V0}}\right)^m$ and, so, the chance that this *will not* happen, and that any party will win *at least one single* seat, must then be:

$$\Pr(m_i \geq 1) = 1 - \left(1 - \frac{1}{n_{V0}}\right)^m \tag{6}$$

Where the district magnitude, m, and the number of vote-winning parties, n_{V0}, tend to ∞, the probability that any party will win at least one seat in a district converges on $1 - \frac{1}{e} \approx 0.63$ (where $e \approx 2.71$ is Euler's constant). Further, Figure 1 plots the probabilities from Equation 6 as n_{V0} and m increase in size (i.e. $m = n_{V0} = 1, 2, \ldots, \infty$) and shows that this convergence occurs very quickly, even for low values of n_{V0} and m. So, if the district magnitude and the number of vote-winning parties are sufficiently large, you should expect any given party to have around a 63% chance of winning at least one seat in a district, all else being equal.

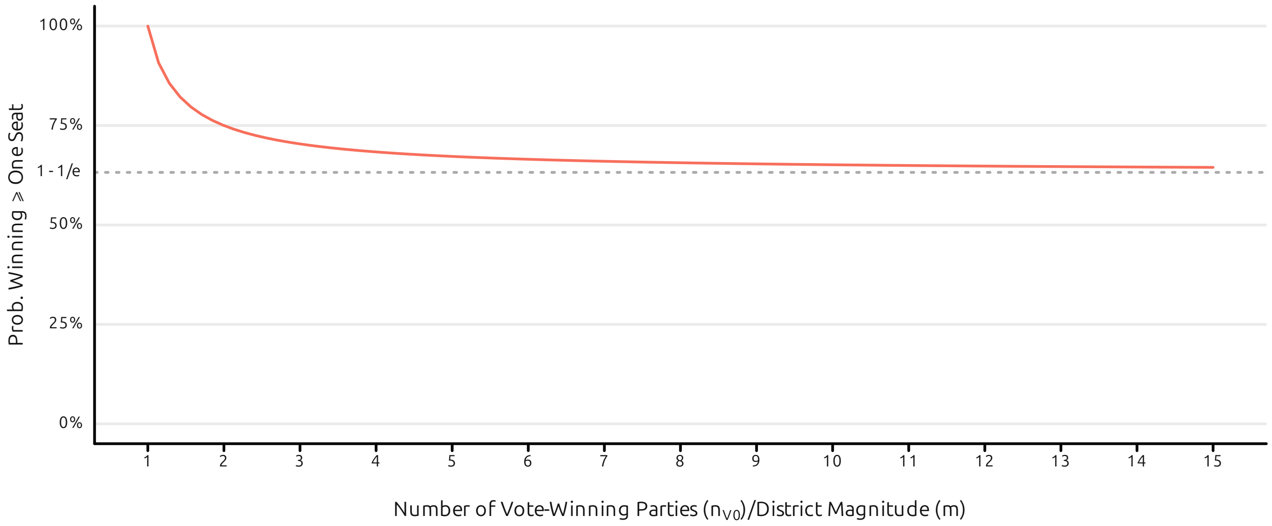

Figure 1: As the district magnitude and the number of vote-winning parties each increase in size (i.e. $n_{V0} = m = 1, 2, \ldots$), the probability that any given party will win at least one seat in a district converges on 1 - 1/e ≈ 0.63, all else being equal. As such, any vote-winning party has a better than even chance of winning a seat in most districted systems of proportional representation.

Clearly, this combinatorial logic is at odds with Lijphart (1994) and Taagepera's (2002) assumption that the thresholds of representation and exclusion should have equal weight. Instead, my combinatorial logic suggests that we should assign a weight of $1 - \frac{1}{e} \approx 0.63$ to the threshold of exclusion, t_X, since it reflects the point at which a party is *guaranteed* to win its first seat in a district, and that, for the opposite reason, we should assign a weight of $1 - \left(1 - \frac{1}{e}\right) \approx 0.37$ to the threshold of representation, t_R, for the same reason. If we update Lijphart's (1994) model, shown in Equation 4, to account for this expectation, we arrive at:

$$t = \frac{1 - \left(1 - \frac{1}{e}\right)}{mn_{V0}} + \frac{1 - \frac{1}{e}}{m + 1} \tag{7}$$

Which simplifies to give my logical model of the effective district-wide threshold:

$$t = \frac{1}{emn_{V0}} + \frac{e - 1}{e(m + 1)} \tag{8}$$

3.2 EMPIRICAL CORROBORATION

My logical model represents a best guess at the effective district-wide threshold *in the absence of all other data*. As such, reality might object to the logic that I set out above. To test this, I use district-level data from the Constituency-Level Elections Archive (Kollman et al. 2024). After removing errant cases—for example, where districts have a magnitude of zero—and subsetting to country-years that score 0.5 or above on V-Dem's (Coppedge et al. 2025) polyarchy index in simple electoral systems (Shugart and Taagepera 2017), I am left with 91,131 unique observations, each representing some party in some district at one of 198 elections in one of 60 different countries between 1946 and 2022.

Any model capable of testing my logic must have at least two parameters. First, it must have a slope parameter, β, to allow the model to discriminate between cases where a party did ($w_i = 1$) or did not ($w_i = 0$) win at least one seat in a district. Second, it must have a threshold parameter, t, that estimates the effective district-wide threshold. The simplest such model is the two-parameter logistic function, which, if we let v_i represent the vote share of some party i, we can write as follows:

$$w_i \sim \text{Bernoulli}(\pi_i) \tag{9}$$

$$\text{logit}(\pi_i) = \beta\,(v_i - t) \tag{10}$$

At present, this model can make impossible predictions. For example, where the threshold parameter, t, is low, which we might expect to be the case in districts with large magnitudes and many vote-winning parties, it could predict that parties still have a chance of winning a seat even where their vote share is 0%. Thankfully, accounting for this is straightforward: we need only put each party's vote share, v_i, and the model's estimate of the effective district-wide threshold, t, on the logit scale by taking their log odds. Integrating this change gives:

$$w_i \sim \text{Bernoulli}(\pi_i) \tag{11}$$

$$\text{logit}(\pi_i) = \beta\,[\text{logit}\,(v_i) - \text{logit}\,(t)] \tag{12}$$

Another limitation that the model has is that it estimates only a single effective district-wide threshold. However, we know from Equation 7 that this threshold should depend on the district magnitude, m, and the number of vote-winning parties, n_{V0}. As such, rather than estimate t from the data, we can model it directly by substituting in Equation 7 (which I express in terms of the bounding thresholds to reduce visual clutter) with one small change: rather than include $1 - \frac{1}{e}$ in the numerator, we include θ, a weighting parameter that we estimate from the data. If my logic is correct, θ should equal around $1 - \frac{1}{e} \approx 0.63$. This gives:

$$w_i \sim \text{Bernoulli}(\pi_i) \tag{13}$$

$$\text{logit}(\pi_i) = \beta\,[\text{logit}\,(v_i) - \text{logit}\,(t_i)] \tag{14}$$

$$t_i = (1 - \theta)\,t_R + \theta t_X \tag{15}$$

We also know that the discrimination parameter, β, must vary depending on the district magnitude, m, and the number of vote-winning parties, n_{V0}, as well. This occurs because the bounding thresholds, t_R and t_X, differ from case to case as a function of these variables. As such, the value that β takes must change to accommodate the distance that emerges between them. The *smallest* distance between the bounding thresholds occurs where $m = 1$ and $n_{V0} = 2$. Here, the thresholds of representation and exclusion both equal 50% and so there is no distance

Table 1: My argument suggests that the mixing parameter, θ, should equal around 1 - 1/e ≈ 0.63. Remarkably, this is exactly the case, suggesting that the logic that I set out is correct. Data here come from district-level election results from the Constituency-Level Elections Archive.

	Estimate	Error	2.5%	97.5%
Discrimination, λ	8.78	0.093	8.60	8.96
Weight, θ	0.63	0.0026	0.63	0.64
Bayesian R^2	0.79	0.00072	0.79	0.79
Observations	91,131			
Elections	198			
Countries	60			

between them. The *largest* distance, instead, occurs where $m = 1$ and n_{v0} tends to infinity. In this case, the threshold of exclusion still equals 50%, but the threshold of representation converges on 0%, resulting in a 50 percentage point distance.

This suggests that we should also model the discrimination parameter, β, as a function of the distance between the bounding thresholds of representation, t_R, and exclusion, t_X. To do so, recall that, where $m = 1$ and $n_{v0} = 2$, both the threshold of representation, t_R, and the threshold of exclusion, t_X, equal 50%. As such, the slope here must be infinite to allow for a step change in the probability of winning a seat that goes from 0% to 100% at the electoral threshold. With this in mind, now imagine some new parameter, λ, that represents a near-100% probability, measured in units of logits. If we now set $v_i = t_X$ and $t_i = t_R$, we can then rearrange to get:

$$\beta = \frac{\lambda}{\text{logit}\,(t_X) - \text{logit}\,(t_R)} \tag{16}$$

Which we can substitute into our previous non-linear model to get:

$$w_i \sim \text{Bernoulli}(\pi_i) \tag{17}$$

$$\text{logit}(\pi_i) = \beta\,[\text{logit}\,(v_i) - \text{logit}\,(t_i)] \tag{18}$$

$$\beta = \frac{\lambda}{\text{logit}\,(t_X) - \text{logit}\,(t_R)} \tag{19}$$

$$t_i = (1 - \theta)\,t_R + \theta t_X \tag{20}$$

Though this model involves many complex constraints, it still has only two parameters: the discrimination parameter, λ, which represents a probability of 100% measured in units of logits and the weighting parameter, θ, which determines the extent to which we favour the threshold of exclusion, t_X, over the threshold of representation, t_R.[4] Due to the logic that I set out in Section 3.1, we already have a theoretical value for θ: $1 - \frac{1}{e} \approx 0.63$. But we can also set similar expectations for λ as well. Note that, in principle, λ should be *infinitely* large, since

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[4] Note that I also include weakly-informative priors on these two parameters. For more information, including all relevant code, see the replication package that accompanies this note.

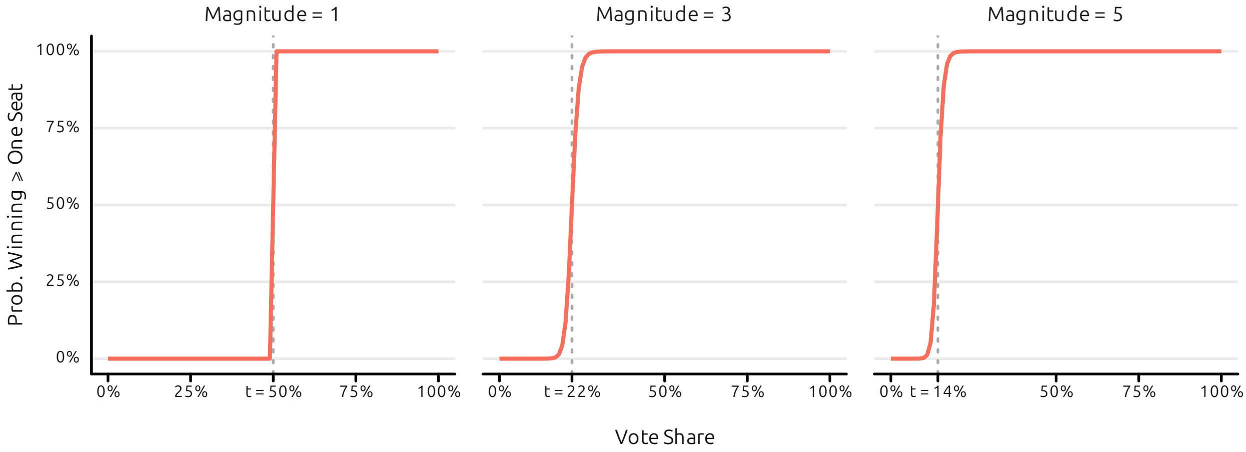

Figure 2: If we keep the number of vote-winning parties constant, in this case setting it equal to 2, and increase the district magnitude, the effective district-wide threshold decreases in turn, thereby causing the electoral system to become much more permissive.

where $t_R = t_X = 0.5$, the logistic curve must become a step function which, in turn, means that β must be infinitely large. In practice, however, statistical algorithms tend not to work well with infinities and handle only a limited sample of data. As such, rather than expect $\lambda = \infty$, we might expect it to equal 5 logits or more, since $\text{logit}^{-1}(5) \approx 99\%$.

Table 1 corroborates my logic in both cases. Remarkably, the weighting parameter, θ, is *exactly* equal to my theoretical expectation of $1 - \frac{1}{e} \approx 0.63$ to two significant figures. Likewise, the discrimination parameter, λ, also takes a large value, here exceeding my expectation of around 5 logits, to allow the function to discriminate between cases where $t_X = t_R = t$ and winning a seat is no longer probabilistic. Similarly, the model also fits well to the data and has a Bayesian R^2 of 0.79, suggesting that it explains a good deal of the variation in the probability that a given party will win at least one seat in a district. As the predictions in Figure 2 make clear, the model also behaves much as we would expect. In this case where, the number of vote-winning parties is fixed such that $n_{V0} = 2$, as the district magnitude increases from $m = 2$ through $m = 3$ to $m = 5$, the effective district-wide threshold becomes much more permissive, decreasing from 50% to 22% to 14%. Likewise, Figure 2 also makes clear that the model's discrimination between cases with a high and low probability of winning representation is relatively steep, especially as the district magnitude grows. So, while the effective district-wide threshold is not a *sharp* discontinuity, it might still not be overly fuzzy.

Since we have strong empirical evidence to corroborate both that the weighting parameter $\theta = 1 - \frac{1}{e}$ and that the discrimination parameter λ must be a large enough number of logits to approximate a probability of 100% (say, $\text{logit}^{-1}(9) \approx 99.99\%$ if we round the empirical estimate in Table 1 to the nearest figure), we can use this information to define a logical model of the propensity for any given party to win at least one seat in some district under either first-past-the-post or districted proportional representation, which we can write as follows:[5]

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[5] To make it as easy as possible for others to implement these logical models in their own research, I define a series of convenience functions in the R programming language in Section A in the appendix to this paper.

$$w_i = \operatorname{logit}^{-1}(\pi_i) \tag{21}$$

$$\operatorname{logit}(\pi_i) = \beta \left[\operatorname{logit}(v_i) - \operatorname{logit}(t_i)\right] \tag{22}$$

$$\beta = \frac{9}{\operatorname{logit}(t_X) - \operatorname{logit}(t_R)} \tag{23}$$

$$t = \frac{1}{emn_{V0}} + \frac{e-1}{e(m+1)} \tag{24}$$

$$t_R = \frac{1}{mn_{V0}},\ t_X = \frac{1}{m+1} \tag{25}$$

4 CONCLUSION

In this short note, I have set out to answer a seemingly simple question: where does the “post” lie in first-past-the-post and other “simple” electoral systems? To do so, I have derived a logical model of the relationship between a district’s magnitude, m; the number of vote-winning parties that it contains, n_{V0}; and its effective threshold, t: the point at which any given party has a 50/50 chance of winning its first seat in the district. Though real world data corroborate my logic exactly, its derivation does not rely on data alone. Rather, it is based only on simple combinatorics. Armed with compelling evidence in favour of my assumptions, I then derived a related logical model that builds on the first to predict the propensity of any party to win at least one seat in a district.

The two logical models that I present here serve to advance the broader Taageperan research agenda by providing more parsimonious and interlocking predictive models of parameters inherent to electoral systems. Logically, future research should proceed in two directions. First, it should seek to establish the equivalent quantity at the national level. Second, it should use the interlocking nature of these models to explore how factors that predict district magnitudes and the number of vote-winning parties affect party systems by by altering effective electoral thresholds.

My findings also have the potential to unlock new causal designs. For example, future research might use the model that I specify in Equations 22 to 25 to compute propensity scores that they can condition on to remove bias from potential confounders and recover causal effects. And, though propensity scores methods are somewhat maligned since we almost never know the true propensity score given a pattern of covariates (King and Nielsen 2019), that is not the case here, since the true propensity is determined almost exactly by only n_{V0} and m. Similarly, much like recent research by Matsunaga and Winzen (2024), Valentim and Dinas (2024), and others who use Taagepera’s (2002) model to exploit the quasi-random assignment to seat-winning parties that occurs around electoral thresholds, future research might use the thresholds that I estimate to specify new fuzzy regression discontinuity designs that explain the impact that assigning new parties to some district have on other electoral factors.

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A R FUNCTIONS

```r
# Convert probabilities to logits

logit <- function(p){ log(p) - log1p(-p) }


# Convert logits to probabilities

inv_logit <- function(x){ 1/(1 + exp(-x)) }


# Compute the threshold of representation

tR <- function(m, n){ 1 / ( m * n ) }


# Compute the threshold of exclusion

tX <- function(m){ 1 / ( m + 1 ) }


# Compute the effective district-wide threshold

effective_threshold <-
  function(m, n){

    # Define weight

    wt <- 1 - 1 / exp(1)


    # Return effective district-wide threshold

    ( 1 - wt ) * tR(m, n) + wt * tX(m)

  }
```

```
# Compute propensity of winning at least one seat

seat_propensity <-
  function(v, m, n){

    # Compute beta

    b <- 9 / ( logit( tX(m) ) - logit( tR(m, n) ) )


    # Compute effective threshold

    t <- effective_threshold(m, n)


    # Return propensity

    inv_logit( b * ( logit(v) - logit(t) ) )

  }
```